The Poppy Academy Trust Calculation Policy 2022



St. John's Nursery and Infant School Fair Field Junior School

EYFS - Year 6 Calculation Policy

Introduction

This Calculation Policy has been produced in line with the 2014 National Curriculum for Mathematics to ensure consistency and progression in teaching throughout the school that is age appropriate. It aims to introduce children to the processes of calculation through concrete, pictorial and abstract activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases and learn to interpret and use the signs and symbols involved. This policy shows the natural progression that a child should make in their mathematical education. Children should not progress onto the advanced stages of formal written methods until they have a secure conceptual understanding.

Intent

Maths is a journey and long-term goal, achieved through exploration, clarification, practice and application over time. At each stage of learning, children should be able to demonstrate a deep, conceptual understanding of the topic and be able to build on this over time.

Our overall aims for when children leave St. John's Nursery and Infants and Fair Field Junior School are:

- we ensure children feel confident to apply their skills to everyday life and understand that maths is all around us and intrinsic to a successful life.
- develop a positive attitude to mathematics as a subject in which all children gain success and pleasure.
- have access to a high quality maths curriculum that is both challenging and enjoyable, and builds upon previous learning.
- be provided with a variety of mathematical opportunities, which will enable them to make relevant connections.
- ensuring children are confident mathematicians who are not afraid to take risks.
 - children are confident when reasoning and make appropriate decisions, applying mathematical thinking in order to solve problems.
- develop an ability to express themselves fluently, to talk about the subject with assurance, using correct mathematical language and vocabulary.
- develop mathematical skills and knowledge and recall of basic number facts and the four operations
- be able to use this knowledge and understanding to carry out calculations mentally
- make use of pictorial representations and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads

• have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally.

They will do this by always asking themselves: Can I do this in my head? Can I do this in my head using pictorial representations? Do I need to use a pencil and paper procedure of a formal written method?

Implementation

The school uses Herts For Learning 'ESSENTIAL Maths' as the scheme of work that provides continuity and progression ensuring appropriate pitch and coverage of the curriculum. We continue to modify to match with our school's approach and the needs of our pupils.

This policy is a statement of the aims, principles and strategies for teaching and learning of calculation strategies in Mathematics. It is designed to help teachers and staff at St. John's Nursery and Infant School and Fair Field Junior School ensure that calculation is taught consistently across the school and to aid them in helping children who may need extra support or challenges. This policy is also designed to help parents, carers and other family members support children's learning by providing an explanation of the methods used in our school.

The policy is set out according to the 4 operations, addition, subtraction, multiplication and division; and incorporates fractions for KS2. Within each specific area there is a progression of skills, knowledge and layout for written methods. The calculation strategies which will be used will reflect this ideology – moving from concrete to pictorial and then abstract recording leading to more formal written methods. The expectations are that teachers follow this to ensure consistency in approach and delivery at the appropriate stage. Mental methods and strategies will work in partnership with these methods. A variety of mental calculation methods will be taught and that recall of facts will be taught in school and tested regularly. The progression of mental methods and expectations will comply with the National Curriculum Statements 2014. It is important staff throughout the trust use the correct mathematical language and encourage pupils to do this also, which will help them develop confidence in their mathematical processes and reasoning. This will take place in class discussions as well as through oral and written feedback, next steps and target setting. The basis of our maths calculation policy is that written methods are complementary to mental methods and should not be seen as separate from them.

The Importance of Mental Mathematics

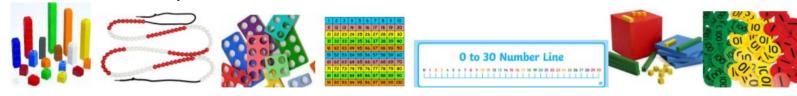
While this policy focuses on written calculation in mathematics, we recognise the importance of mental strategies and known facts

that form the basis of all calculations. Pupils are provided with frequent opportunities to compare and evaluate different calculation strategies. This helps them develop an understanding that efficiency is personal and based on the numbers involved. Mental maths fluency underpins all effective written calculation approaches.

Concrete, Pictorial and Abstract

Concrete - Manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways. All pupils should have frequent opportunities to develop their understanding of mathematical concepts through the appropriate use of concrete apparatus.

Concrete resources that may be found in classrooms will include:



These resources will vary depending on year group and individual needs. At home, pupils very well may not have access to these school resources; however, they are just a vehicle to support a pupil's understanding of a topic. Any objects can be used at home to replace counters, cubes etc.

Pictorial (including jottings) - The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives. When a child is working at the pictorial stage, it often provides rich opportunities for assessment of their depth of understanding.

Abstract - **Written** The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

Abstract - **Spoken Learning** to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using speaking frames.

Impact

Pupils will leave us prepared for the next stage in their lives with:

- Pupils have an appreciation for the maths in everyday life
- Quick recall of facts and procedures
- The flexibility and fluidity to move between different contexts and representations of mathematics
- The ability to recognise relationships and make connections in mathematics
- Confidence and belief that they can achieve
- The knowledge that maths underpins most of our daily lives
- Skills and concepts that have been mastered
- Have a positive and inquisitive attitude to mathematics as an interesting and attractive subject in which all children gain success and pleasure.

A mathematical concept or skill has been mastered when a child can show it in multiple ways, using the mathematical language to explain their ideas, and can independently apply the concept to new problems in unfamiliar situations and this is the goal for our children. These will be assessed through: assessment, tracking, pupil progress meetings, performance management, moderation and standardisation.

Early Years Foundation Stage

Children at the expected level of development will:

- Have a deep understanding of number to 10, including the composition of each number
- Subitise (recognise quantities without counting) up to 5
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Children are encouraged to gain a sense of the number system through the use of counting concrete objects.

Addition





ways and count all.



They understand addition as counting on and will count on in ones and twos using object s, cubes, bead string and number line.



They use concrete and pictorial representation to record their calculations.

When confident children may be able to pictorially represent their calculations using symbols and

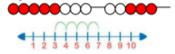
Children are encouraged to gain a sense of the number system through the use of counting concrete objects

and understand subtraction as counting out.

Subtraction



They begin to count back in ones and twos using objects, cubes, bead string and number line.



They may use concrete and pictorial representation to record their calculations.

They are encouraged to develop a mental picture of the number system in their heads to use for calculations. When confident children may be

Children use concrete objects to make and count equal groups of

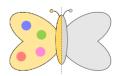
Multiplication

objects.



They will count on in twos using a bead string and number line.

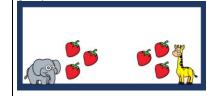
They understand doubling as repeated addition. 4+4 =8



They use concrete and pictorial representation to record their calculations. When confident

Division

Children use concrete objects to count and share equally into 2 groups.



They count a set of objects and halve them by making two equal groups.

They understand sharing and halving as dividing by 2.



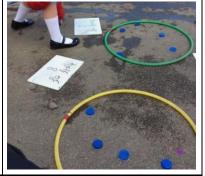
numbers within a written

able to represent their calculations using symbols and numbers within a written calculation.

children may be able to represent their calculations using symbols and numbers within a written calculation.



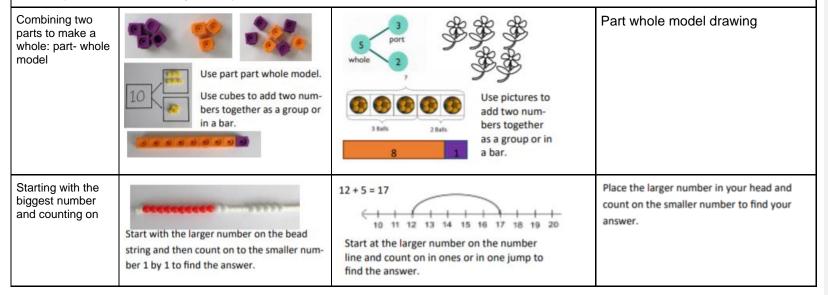
They will begin to use objects to make groups of 2 from a given amount. They use concrete and pictorial representation to record their calculations.



Year 1 Addition

Pupils should be able to:

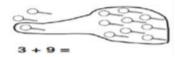
- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 7 = –9.



Regrouping to make 10. (This is an essential skill for column addition later).



6 + 5 = 11



nn addition

Start with the bigger number and use the smaller number to make 10. Use ten frames.

Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.

7 + 4 = 11

If I am at seven, how many more do I need to make 10. How many more do I add on now?

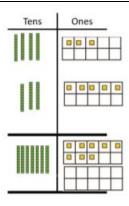
Year 2 Addition

Pupils should be taught to:

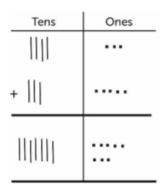
- add numbers using concrete objects, pictorial representations, and mentally including:
 - a two-digit number and ones a two-digit number
 - tens two two-digit numbers
 - adding three one-digit numbers
 - solve problems with addition: using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods
 - recall and use addition facts to 20 fluently, and derive and use related facts up to 100

Objectives and strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 17 + 5 = 22 20 16 + 7 16 + 7	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22-17 = 5 22-5 = 17
Add a 2 digit number and tens	25 + 10 = 35 Explore that the ones digit does not change	27 + 30 +10 +10 +10 	27 + 10 = 37 27 + 20 = 47 27 + \(\sigma\) = 57

Add two 2digit numbers (without regrouping)



Using base-ten, numicon and place value counters to support adding two 2-digit numbers without regrouping.



When confident they will begin to draw the expanded written method.

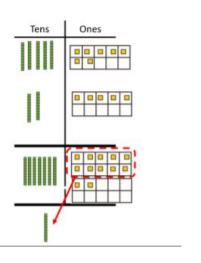
We provide lots of strategies for adding two digit numbers for example number lines -

Adding tens and ones -

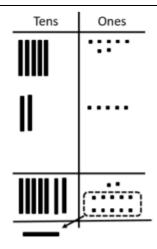
$$43 + 35 = 78$$

When confident they will begin to use the expanded written method to add two digit numbers.

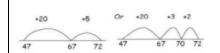
Add two digit numbers with regrouping (through ten)



Add the ones and regroup them on the tens frame to exchange for a ten. Add the tens together.



After practically using the base ten, they will begin to draw using a pictorial representation of the expanded written method (with regrouping).



Use number line and bridge ten using part whole if necessary.

When they are confident they will begin to use the expanded written method to add two two -digit numbers with regrouping.

Children will also have the opportunity to use partitioning for addition.

$$25 + 47$$

$$20 + 5$$

$$40 + 7$$

$$20 + 40 = 60$$

$$5 + 7 = 12$$

$$60 + 12 = 72$$

Add three 1-digit numbers.

Combine to make 10 first if possible, or bridge 10 then add third digit

Combine to make 10 first if possible, or bridge 10 then add third digit

Combine to make 10 first if possible, or bridge 10 then add third digit

Add three 1-digit

Regroup and draw representation.

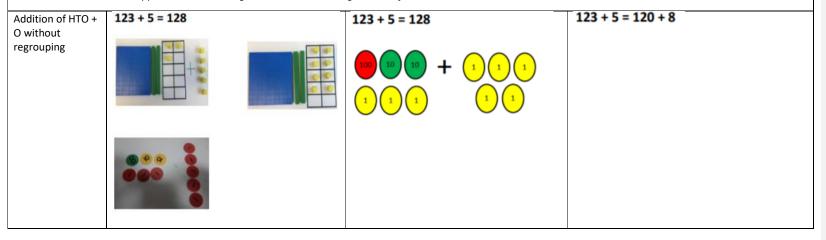
Combine the two numbers that make/bridge ten then add on the third.

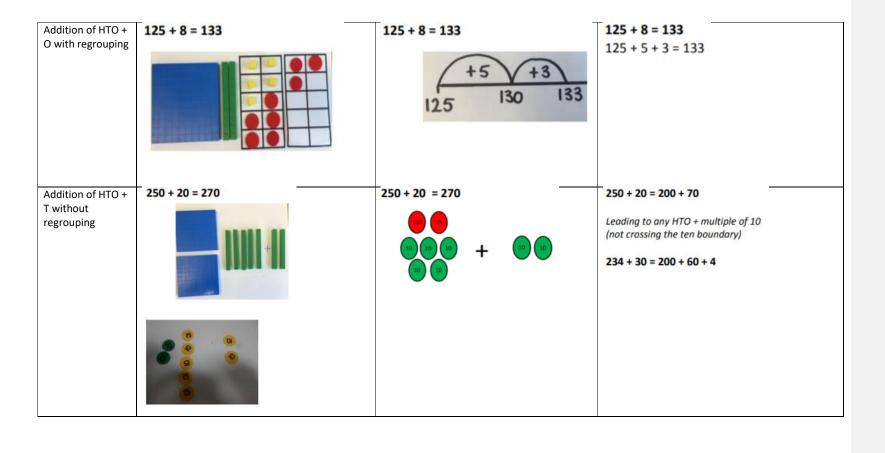
Year 3 - Addition

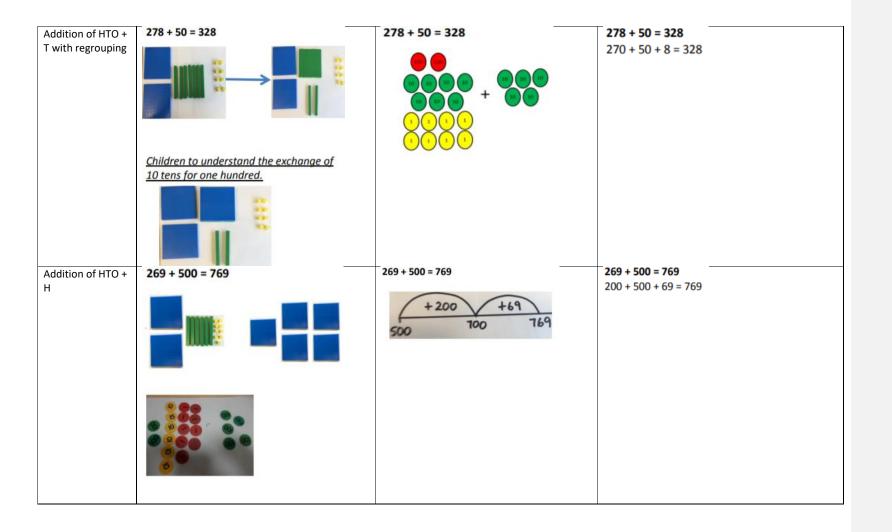
Pupils should be taught to:

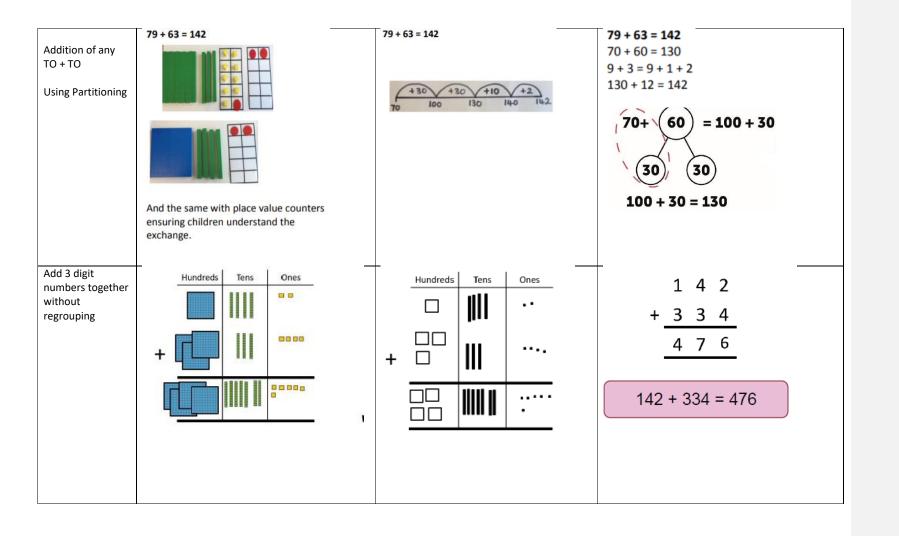
- add numbers mentally, including:
- a three-digit number and 1s
- a three-digit number and 10s
- a three-digit number and 100s
- add numbers with up to 3 digits, using formal written methods of columnar addition
- solve problems, including missing number problems, using number facts, place value, and more complex addition.

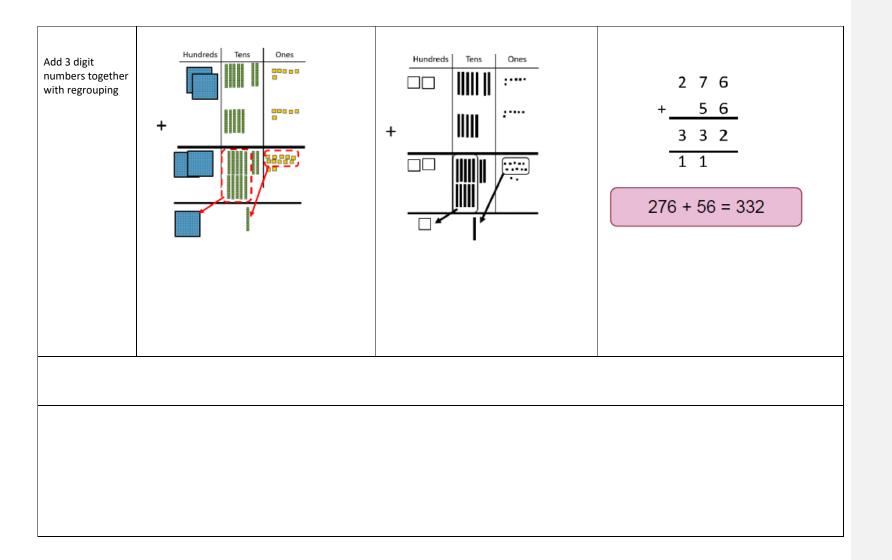
Bar models to be used to support decision making and where the missing numbers fit in our calculations.









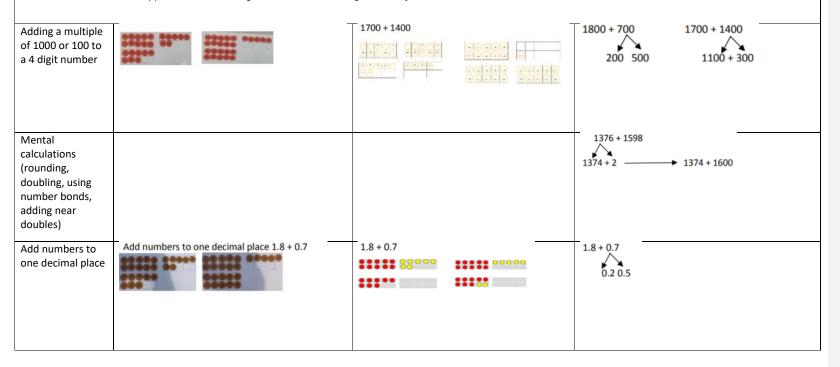


Year 4 - Addition

Pupils should be taught to:

- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why

Bar models to be used to support decision making and where the missing numbers fit in our calculations.



Add numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Use place value equipment on a place value grid to organise thinking.

Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.

Use equipment.to show 1,905 + 775.

Th	н	T	0
•	00000		00000
	00000 00	00000 00	00000

Why have only three columns been used for the second row? Why is the Thousands box empty?

Which columns will total 10 or more?

Use place value equipment to model required exchanges.

Th	Н	T	0	
•	00000	00000	0000	
0000	00	000	00000	
		<u>-</u>		
Th	н	Т	0	
•	00000	00000		
0000	••	000	•	
0				
Th	Н	T	0	
•	00000	00000		
0000	••	000	•	
		•		
Th	Н	T	0	
•	00000	00000		
0000	00	000		
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Include examples that exchange in more than one column.

Use a column method to add, including exchanges.

Include examples that exchange in more than one column.

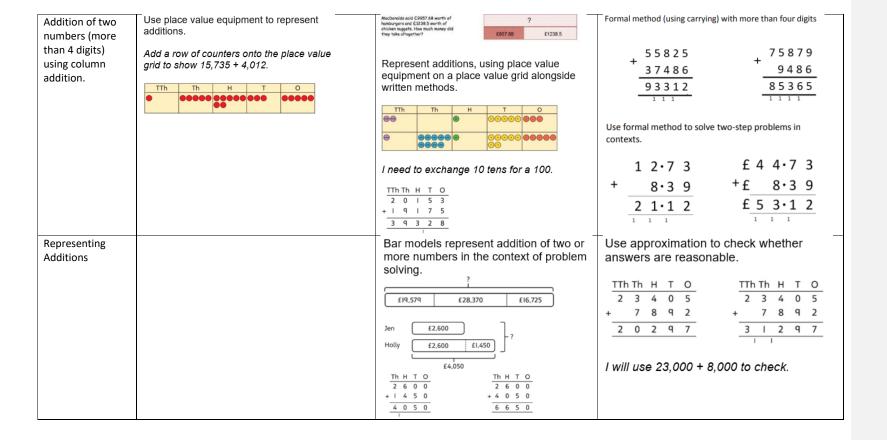
Addition Year 5

Pupils should be taught to:

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Add numbers mentally with increasingly large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Bar models to be used to support decision making and where the missing numbers fit in our calculations.

Mental	Examples
calculations	
(rounding,	1445 + 2999
doubling, using	1445 + 3000 – 1
number bonds,	1200 : 1200
adding near	1299 + 1299
doubles)	Double 1300 - 2
doubles	
	443 + 445
	Near double 443 + 2
	12.36 + 5.24
	0.36 + 0.24 = 0.6
	17 + 0.6 = 17.6
	36.25 + 23.43
	Add each place value column individually



Adding Tenths	Link measure with addition of decimals. Two lengths of fencing are 0.6 m and 0.2 m. How long are they when added together? 0.6 m 0.2 m	Use a bar model with a number line to add tenths. 0.6 m 0.2 m 0.1 m 0.2 m 0.3 m 0.4	Understand the link with adding fractions. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ $6 \text{ tenths} + 2 \text{ tenths} = 8 \text{ tenths}$ $0.6 + 0.2 = 0.8$
Adding decimals using column addition	Use place value equipment to represent additions. Show 0·23 + 0·45 using place value counters.	Use place value equipment on a place value grid to represent additions. Represent exchange where necessary. O Tth Hth O Q 2 O 3 3 O 3 1 O 2 5 Include examples where the numbers of decimal places are different. O Tth Hth O O Tth Hth	Add using a column method, ensuring that children understand the link with place value. $ \frac{O \cdot \text{Tth Hth}}{0 \cdot 2 \cdot 3} + \frac{0 \cdot 4 \cdot 5}{0 \cdot 6 \cdot 8} $ Include exchange where required, alongside an understanding of place value. $ \frac{O \cdot \text{Tth Hth}}{0 \cdot 9 \cdot 2} + \frac{0 \cdot 3 \cdot 3}{1 \cdot 2 \cdot 5} $ Include additions where the numbers of decimal places are different. $ 3.4 + 0.65 = 2 $ $ \frac{O \cdot \text{Tth Hth}}{3 \cdot 4 \cdot 0} + \frac{0 \cdot 6 \cdot 5}{0 \cdot 6 \cdot 5} $

Addition Year 6

Pupils should be taught to:

- perform mental calculations, using increasingly large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition multi-step problems in contexts, deciding which methods to use and why solve problems involving addition

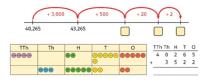
Bar models to be used to support decision making and where the missing numbers fit in our calculations

Comparing and selecting efficient methods

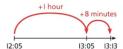
Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.



Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.



Use bar model and number line representations to model addition in problem-solving and measure contexts.



Use column addition where mental methods are not efficient. Recognise common errors with column addition.

Which method has been completed accurately?

What mistake has been made?

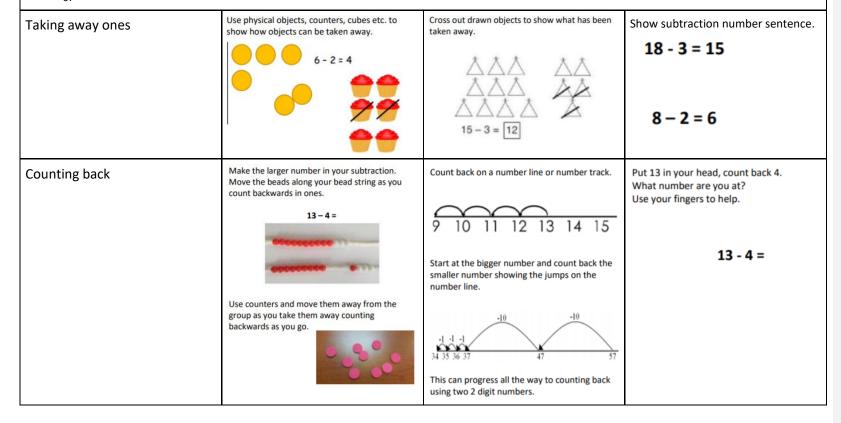
Column methods are also used for decimal additions where mental methods are not efficient.

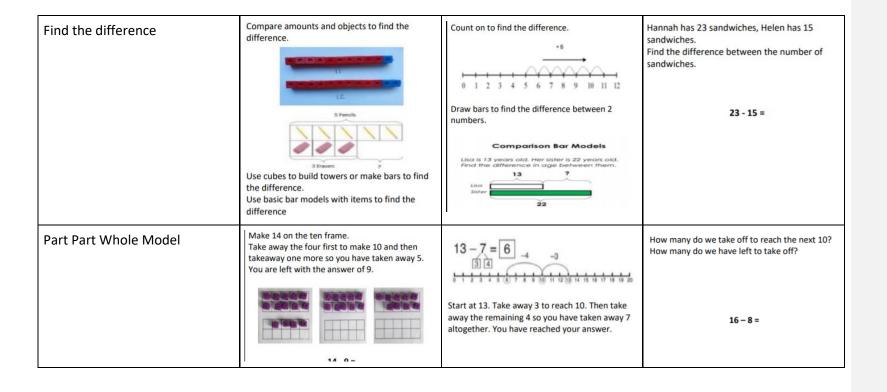
Selecting mental methods for larger numbers where appropriate	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods. 2,411,301 + 500,000 = ? This would be 5 more counters in the HTh place. So, the total is 2,911,301. 2,411,301 + 500,000 = 2,911,301	Use a bar model to support thinking in addition problems. 257,000 + 99,000 = ? £257,000 £100,000 I added 100 thousands then subtracted 1 thousand. 257 thousands + 100 thousands = 357 thousands 257,000 + 100,000 = 357,000 357,000 - 1,000 = 356,000 So, 257,000 + 99,000 = 356,000	Use place value and unitising to support mental calculations with larger numbers. $195,000 + 6,000 = 2$ $195 + 5 + 1 = 201$ $195 \text{ thousands} + 6 \text{ thousands} = 201$ 195 thousands So, $195,000 + 6,000 = 201,000$
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. $3 \times 5 - 2 = ?$	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations.	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. $4 + 6 \times 16$ $4 + 96 = 100$ $(4 + 6) \times 16$ $10 \times 16 = 160$

Year 1 Subtraction

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = -9.



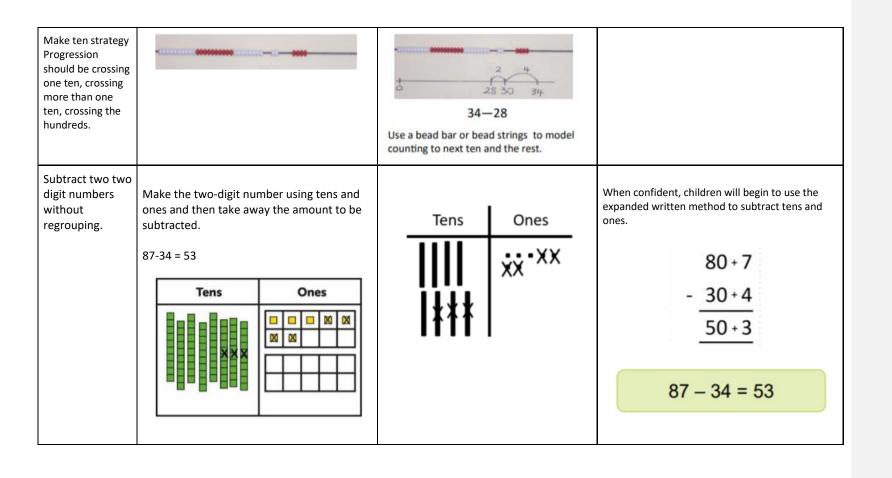


Year 2 Subtraction

Pupils should be able to:

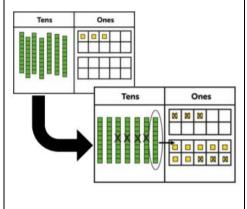
- solve problems with subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers

Objectives and strategies	Concrete	Pictorial	Abstract
Regroup a ten into ten ones and subtract.	1 ten 10 ones =	13-5= 8	13 - 5 = 8

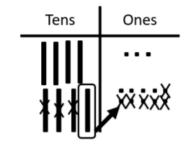


Subtract two digit numbers with regrouping.

Make tens and ones and then regroup one ten for ten ones to be able to subtract the number.



Children will assess if they need to use regrouping to solve the two digit subtraction question. Draw the method they need to use.



When confident, children will begin to use the expanded written method to subtract tens and ones.

$$\begin{array}{r}
60 & 13 \\
70 + 3 \\
40 + 6 \\
\hline
20 + 7
\end{array}$$

$$73 - 46 = 27$$

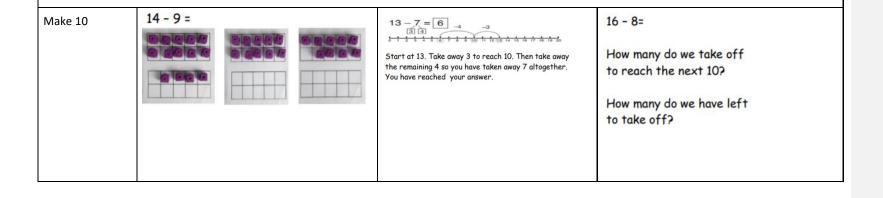
Subtraction Year 3

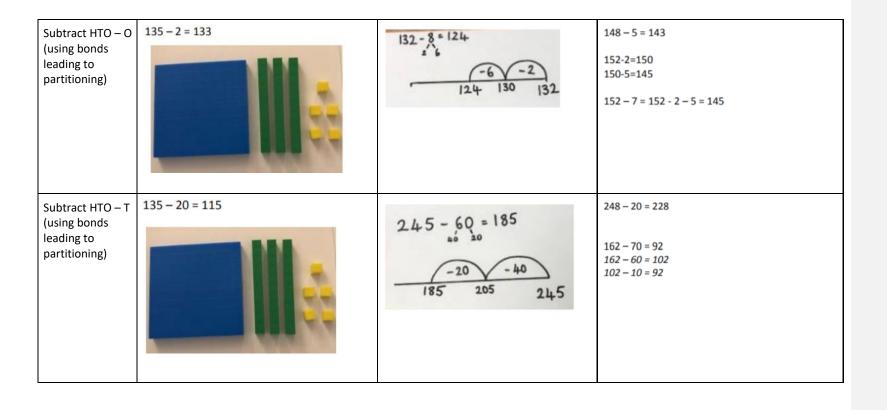
Pupils should be taught to:

- subtract numbers mentally, including:
- a three-digit number and 1s
- a three-digit number and 10s
- a three-digit number and 100s
- subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction

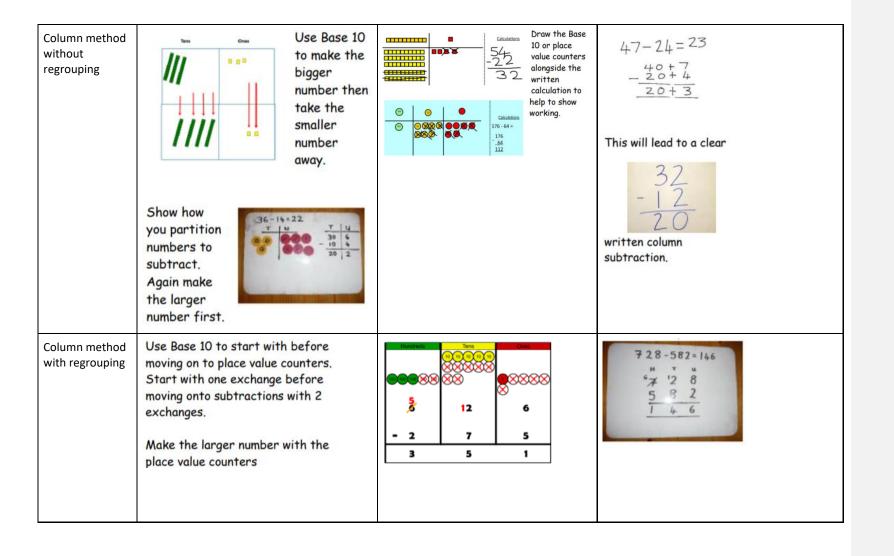
Children should also be taught to calculate the difference when two numbers are close in range e.g. 114 - 98, counting on 98 + 2 = 100 then 100 + 14 = 114, therefore the difference is 16 at all stages.

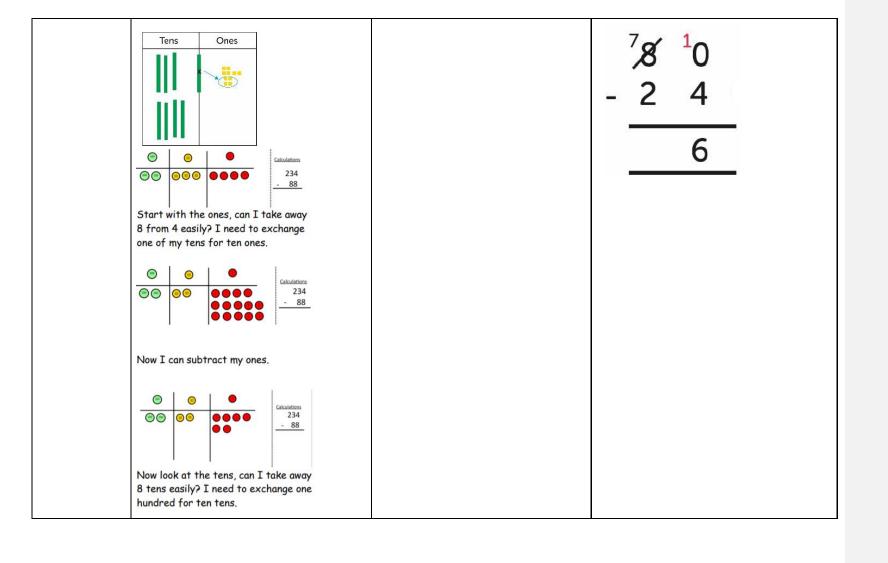
Bar models to be used to support decision making and where the missing numbers fit in our calculations.



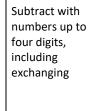


Subtract HTO – H (using bonds)	635 - 400 = 235	$742 - 300 = 442$ $\begin{array}{r} -300 \\ \hline 442 \\ \hline \end{array}$	478 – 200 = 278
Subtract any TO – TO Using partitioning	72 – 26 = 46 Use dienes and place value counters to model 72 – 20 – 2 – 4 = 46	$91-35 = 56$ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	78 - 49 = 29 78 - 40 = 38 38 - 8 = 30 30 - 1 = 39 78 - 40 - 8 - 1 = 29

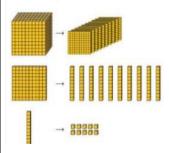




Subtraction Year 4 Pupils should be taught to: • subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve subtraction two-step problems in contexts, deciding which operations and methods to use and why **Bar models to be used to support decision making and where the missing numbers fit in our calculations.** Subtraction of multiples 10/100/1000 **Description of multiples 10/100/1000** **Description of multiples 10/1000** **Description of multiples 10



Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.



Represent place value equipment on a place value grid to subtract, including exchanges where needed.

Th	Н	T	0
9	00	00000	
Th	Н	T	0
9	00	00000	
Th	Н	T	0
	90000	@@@@@	
Th	Н	т	0
	00000	00000	

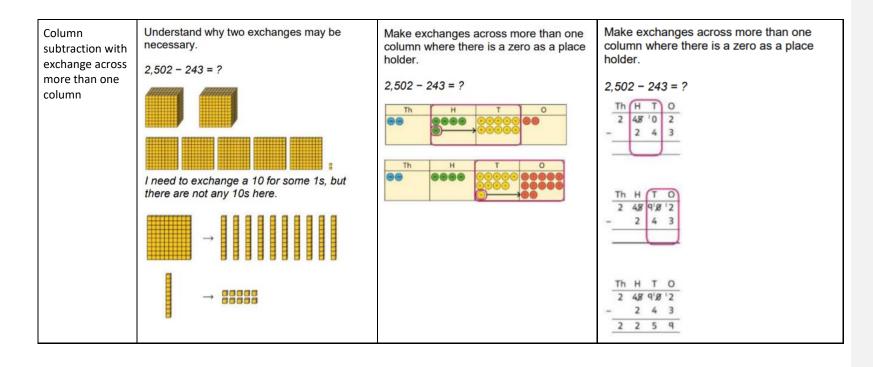
Use column subtraction, with understanding of the place value of any exchange required.

	Th	Н	T	0
	1	2	5	0
-		4	2	0
				0

	Th	Н	T	0
	1	2	5	0
-		4	2	0
			3	0

Th H	Т	0
Y 12	5	0
4	2	0
8	3	0

	$\overline{}$			
1	Th	н	Т	0
	Y	12	5	0
-		4	2	0
		8	3	0



Use bar models to represent subtractions Representing Use inverse operations to check where a part needs to be calculated. subtractions. subtractions and checking Total 5,762 I calculated 1,225 - 799 = 574. strategies I will check by adding the parts. 2.899 Th H T O 7 9 9 I can work out the total number of Yes votes 1,225 + 5 7 4 using 5,762 - 2,899. 799 574 1 3 7 3 Bar models can also represent 'find the difference' as a subtraction problem. The parts do not add to make 1,225. I must have made a mistake. 899 1,005

Subtraction Year 5

Pupils should be taught to:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- solve subtraction multi-step problems in contexts, deciding which methods to use and why

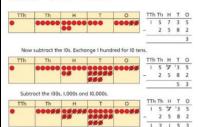
Column subtraction with whole numbers

Use place value equipment to understand where exchanges are required.

2,250 - 1,070



Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.



Use column subtraction methods with exchange where required.

Checking strategies and representing subtractions	Bar models represent subtractions in problem contexts, including 'find the difference'. Athletics Stadium 75,450 Hockey Centre 42,300 Velodrome 15,735	Children can explain the mistake made when the columns have not been ordered correctly. Construction
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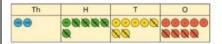
Subtraction Year 6

Pupils should be taught to:

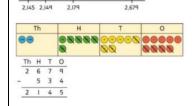
- perform mental calculations, including with increasingly large numbers
- use their knowledge of the order of operations to carry out calculation involving the 4 operations
- solve subtraction multi-step problems in contexts, deciding which methods to use and why
- solve problems using subtraction

Comparing and selecting efficient methods

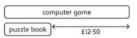
Use counters on a place value grid to represent subtractions of larger numbers.



Compare subtraction methods alongside place value representations.

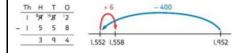


Use a bar model to represent calculations, including 'find the difference' with two bars as comparison.



Compare and select methods.
Use column subtraction when mental methods are not efficient.

Use two different methods for one calculation as a checking strategy.



Use column subtraction for decimal problems, including in the context of measure.

Н	Т	0	*	Tth	Hth
3	0	q	٠	6	0
2	0	6		4	0
1	0	3	٠	2	0

Subtracting mentally with larger numbers Use a bar model to show how unit support mental calculations. 950,000 – 150,000 That is 950 thousands – 150 thousands 950,000 – 150,000 = 800,000	10,000 - 500 = ?
---	------------------

Year 1 Multiplication

Pupils should be taught to:

• solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Doubling	Use practical activities to show how to double a number.	Draw pictures to show how to double a number. Double 4 is 8	Show calculation 3 x 2 = 6
Counting in multiples	Count in multiples supported by concrete objects in equal groups.		Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Repeated addition	Use different objects to add equal groups.		Write addition sentences to describe objects and pictures. $2+2+2+2=10$

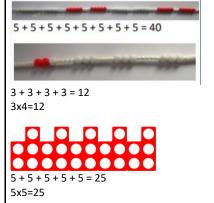
Year 2 Multiplication

Pupils should be able to:

- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers calculate mathematical statements for multiplication a within the multiplication tables and write them using the multiplication (x) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Objective and strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using base ten by partitioning tens and ones. Then double the tens and double the ones. Make the total amount. Double 26: $26 \times 2 =$ $40 + 12 = 52$	Draw pictures and representations to show how to double numbers.	26 x 2

Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition) Using different manipulatives to make repeated additions sums.



Number lines, counting sticks and bar models should be used to show representation of counting in multiples.

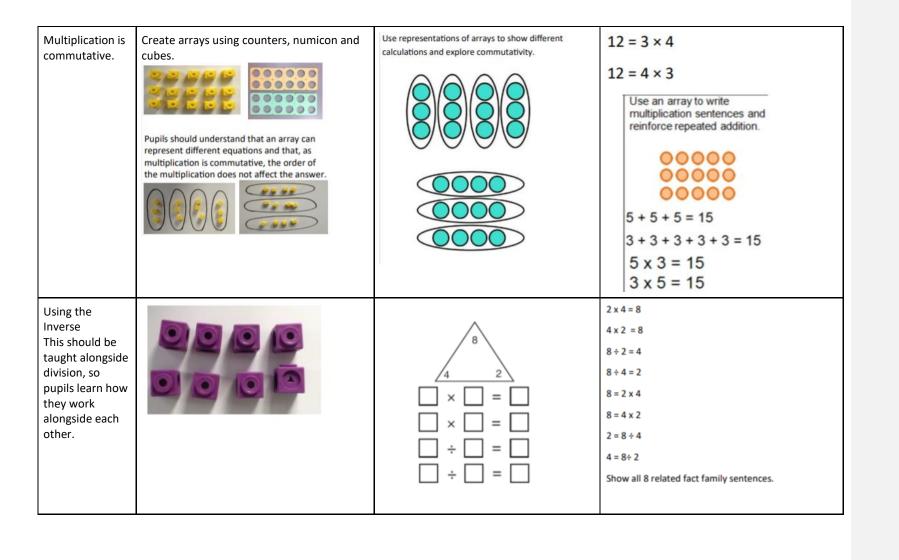
Count in multiples of a number aloud.

Write sequences with multiples of numbers.

0, 2, 4, 6, 8, 10

0, 3, 6, 9, 12, 15

0, 5, 10, 15, 20, 25, 30



Year 3 Multiplication

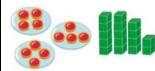
Pupils should be taught to:

- recall and use multiplication facts for the 3,4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in with n objects are connected to m objects

Understanding equal grouping and repeated addition

Children continue to build understanding of equal groups and the relationship with repeated addition.

They recognise both examples and non-examples using objects.



Children recognise that arrays can be used to model commutative multiplications.

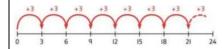


I can see 3 groups of 8. I can see 8 groups of 3.

Children recognise that arrays demonstrate commutativity.



This is 3 groups of 4. This is 4 groups of 3. Children understand the link between repeated addition and multiplication.

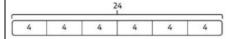


8 groups of 3 is 24.

$$3+3+3+3+3+3+3+3+3=24$$

 $8\times 3=24$

A bar model may represent multiplications as equal groups.



 $6 \times 4 = 24$

Understand how to use times-tables facts Understand how times-table facts relate to Using Understand how times-table facts relate to commutativity. commutativity. commutativity to support I need to work out 4 groups of 7. understanding of the times tables I know that $7 \times 4 = 28$ 00000 so. I know that $6 \times 4 = 24$ 4 groups of 7 = 28 $4 \times 6 = 24$ and 7 groups of 4 = 28. There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls. I can use $6 \times 4 = 24$ to work out both totals. Understanding Children learn the times-tables as 'groups Children understand how the ×2, ×4 and ×8 Children understand the relationship of', but apply their knowledge of tables are related through repeated between related multiplication and division and using ×3, ×2, doubling. commutativity. facts in known times-tables. ×4 and ×8 tables. 000 I can use the ×3 table to work out how ... $2 \times 5 = 10$ 000 many keys. ... 000 000 $5 \times 2 = 10$ I can also use the ×3 table to work out how $10 \div 5 = 2$ $3 \times 2 = 6$ $3 \times 4 = 12$ $3 \times 8 = 24$ many batteries. $10 \div 2 = 5$

Using known facts to multiply 10s, for example 3 × 40

Explore the relationship between known times-tables and multiples of 10 using place value equipment.

Make 4 groups of 3 ones.

000 000 000 000

Make 4 groups of 3 tens.

What is the same? What is different?

Understand how unitising 10s supports multiplying by multiples of 10.

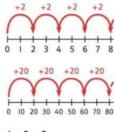




4 groups of 2 ones is 8 ones. 4 groups of 2 tens is 8 tens.

 $4 \times 2 = 8$ $4 \times 20 = 80$

Understand how to use known times-tables to multiply multiples of 10.



$$4 \times 2 = 8$$
$$4 \times 20 = 80$$

Multiplying a 2digit number by a 1-digit number Understand how to link partitioning a 2-digit number with multiplying.

Each person has 23 flowers.

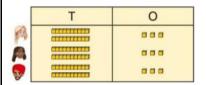
Each person has 2 tens and 3 ones.



There are 3 groups of 2 tens.

There are 3 groups of 3 ones.

Use place value equipment to model the multiplication context.



There are 3 groups of 3 ones.

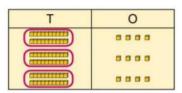
There are 3 groups of 2 tens.

Use place value to support how partitioning is linked with multiplying by a 2-digit number.

 $3 \times 24 = ?$

Т	0
	0000
	000
	(000

 $3 \times 4 = 12$



 $3 \times 20 = 60$

60 + 12 = 72

 $3 \times 24 = 72$

Use addition to complete multiplications of 2-digit numbers by a 1-digit number.

$$4 \times 13 = ?$$

$$4 \times 3 = 12$$
 $4 \times 10 = 40$

$$12 + 40 = 52$$

$$4 \times 13 = 52$$

ı	8		X	3		
Х	ı		0			8
3						
-						
		٠				
		٠				
		٠				
		٠				
		٠			٠	
	1.			_		

The two digit number is partitioned horizontally with the tens digit coming first. The equation is then represented using counters (or an array).



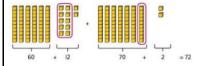
Again, the two digit number is partitioned horizontally with the tens digit coming first. This time the equation is represented using place value counters or Dienes.

Multiplying a 2digit number by a 1-digit number, expanded column method Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications.

 $3 \times 24 = ?$

 $3 \times 20 = 60$

 $3\times 4=12$

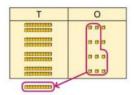


 $3 \times 24 = 60 + 12$ $3 \times 24 = 70 + 2$

 $3 \times 24 = 72$

Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s.

 $4 \times 23 = ?$



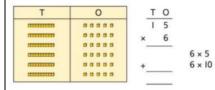
Т	0
	0.0

 $4 \times 23 = 92$

Т	0
<u> </u>	000
00	000
00	000
00	000
00	000

 $5 \times 23 = ?$ $5 \times 3 = 15$ $5 \times 20 = 100$ $5 \times 23 = 115$ Children may write calculations in expanded column form, but must understand the link with place value and exchange.

Children are encouraged to write the expanded parts of the calculation separately.



 $5 \times 28 = ?$

Year 4 Multiplication

Pupils should be taught to:

- recall multiplication facts for multiplication tables up to 12 x 12
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Multiplying by multiples of 10 and 100

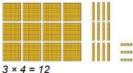
Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



3 groups of 4 ones is 12 ones. 3 groups of 4 tens is 12 tens.

3 groups of 4 hundreds is 12 hundreds.

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



 $3 \times 40 = 120$

 $3 \times 400 = 1,200$

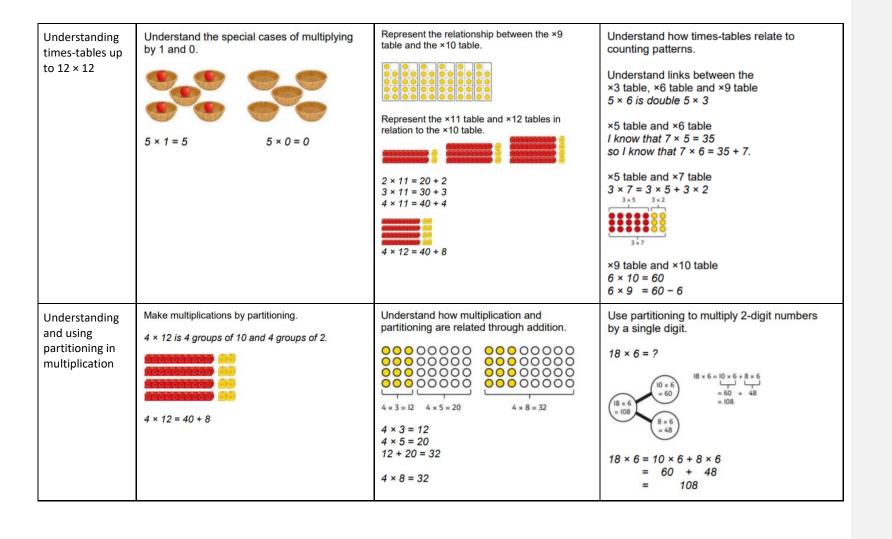
Use known facts and understanding of place value and commutativity to multiply mentally.

$$4 \times 7 = 28$$

$$4 \times 70 = 280$$

 $40 \times 7 = 280$

 $4 \times 700 = 2,800$ $400 \times 7 = 2,800$



Column multiplication for 2- and 3-digit numbers multiplied by a single digit Use place value equipment to make multiplications.

Make 4 × 136 using equipment.



I can work out how many 1s, 10s and 100s.

There are 4 × 6 ones... 24 ones There are 4 × 3 tens ... 12 tens There are 4 × 1 hundreds ... 4 hundreds

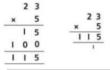
24 + 120 + 400 = 544

Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.



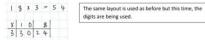
Use the formal column method for up to 3-digit numbers multiplied by a single digit.

Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.



3. Grid method (using place value counters or Dienes)

• TO x O



• HTO x O



The three digit number is partitioned horizontally with the hundreds first followed by the tens and ones.

Multiplying more than two numbers Represent situations by multiplying three numbers together.



Each sheet has 2 × 5 stickers. There are 3 sheets.

There are $5 \times 2 \times 3$ stickers in total.

$$5 \times 2 \times 3 = 30$$
$$10 \times 3 = 30$$

Understand that commutativity can be used to multiply in different orders.



 $2 \times 6 \times 10 = 120$ $12 \times 10 = 120$

$$10 \times 6 \times 2 = 120$$

 $60 \times 2 = 120$

Use knowledge of factors to simplify some multiplications.

$$24 \times 5 = 12 \times 2 \times 5$$

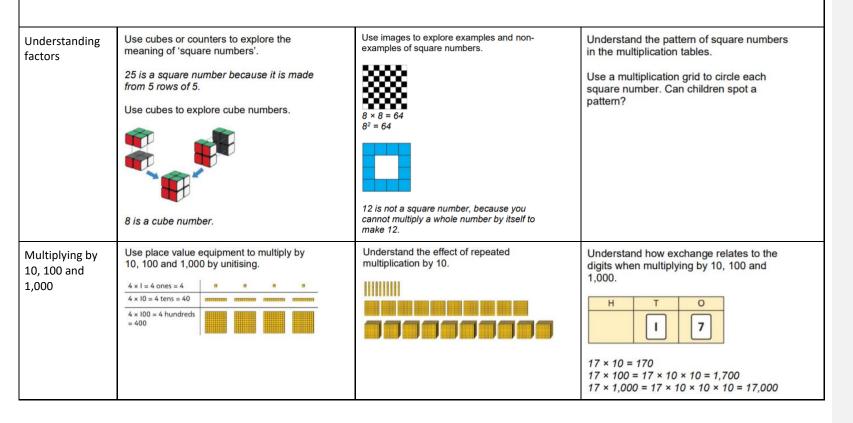
$$12 \times 10 = 120$$

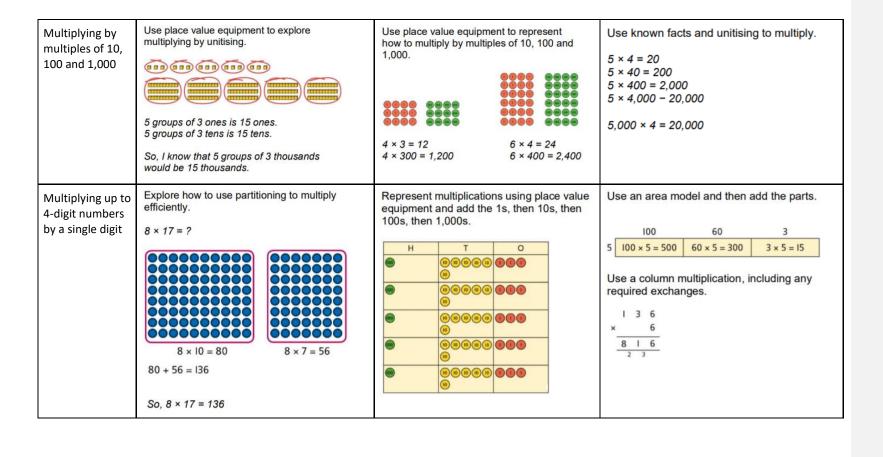
So,
$$24 \times 5 = 120$$

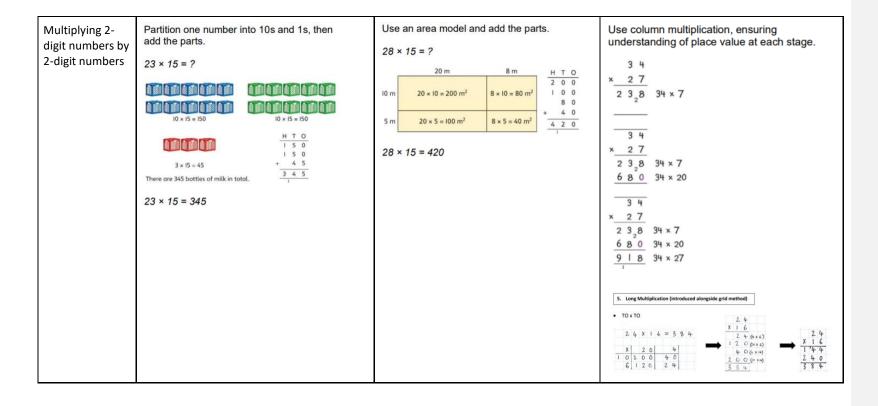
Year 5 Multiplication

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two digit number using a formal written method, including long multiplication for two-digit numbers
- multiply numbers mentally, drawing upon known facts
- multiply whole numbers and those involving decimals by 10,100 and 1,000
- solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving multiplication including understanding the meaning of the equals sign







Multiplying up to 4-digits by 2-digits		Use the area model then add the parts. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Use column multiplication, ensuring understanding of place value at each stage. 1
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Multiplying decimals by 10, 100 and 1,000 Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid. O The Hith Control of the Con	Understand how this exchange is represented on a place value chart. 2.5 × 10 = 25
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Year 6 Multiplication

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operation to carry out calculations involving the 4 operations
- solve problems involving multiplication

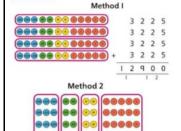
Multiplying up to a 4-digit number by a single digit number Use equipment to explore multiplications.



4 groups of 2,345

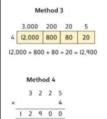
This is a multiplication:

4 × 2,345 2,345 × 4 Use place value equipment to compare methods.

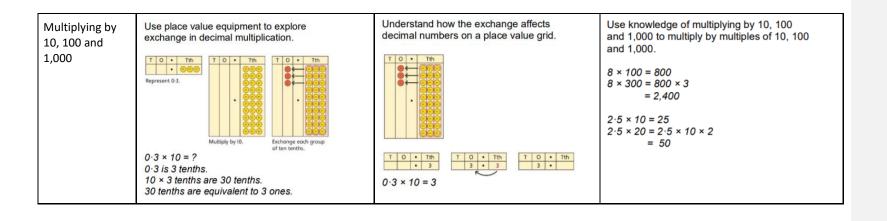


4 × 3.000 4 × 200 4 × 20 4 × 5 12.000 + 800 + 80 + 20 = 12.900 Understand area model and short multiplication.

Compare and select appropriate methods for specific multiplications.



Multiplying up to a 4-digit number by a 2-digit number		Use an area model alongside written multiplication. Method I 1,000 200 30 5 20 20,000 4,000 600 100 1 1,000 200 30 5	Use compact column multiplication with understanding of place value at all stages. 1
		1 2 3 5 1 × 5 3 0 1 × 30 2 0 0 1 × 100 1 1 0 0 1 × 10 1 1 0 1 1 1 1 1 1	- HTO X TO 2 6 2 X 1 9 = 4,9 7 8 X 2 0 0 6 0 2 1 0 2,0 0 0 6 0 0 2 0 2 6 2 0 9 1 8 0 0 5 4 0 1 8 2 3 5 8 9 1 4,9 7 8
Using knowledge of factors and partitions to compare methods for multiplications	Use equipment to understand square numbers and cube numbers. $5 \times 5 = 5^2 = 25$ $5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately. 20 5.200 20 5.200 20 20 20 20 20 20 5.200 25 5	Use a known fact to generate families of related facts.



Multiplying decimals

Explore decimal multiplications using place value equipment and in the context of measures.



3 groups of 4 tenths is 12 tenths. 4 groups of 3 tenths is 12 tenths.



4 × 1 cm = 4 cm 4 × 0·3 cm = 1.2 cm 4 × 1·3 = 4 + 1·2 = 5·2 cm Represent calculations on a place value grid.

$$3 \times 3 = 9$$

$$3 \times 0.3 = 0.9$$

Т	0	•	Tth
		•	

Understand the link between multiplying decimals and repeated addition.





Use known facts to multiply decimals.

$$4 \times 3 = 12$$

 $4 \times 0.3 = 1.2$
 $4 \times 0.03 = 0.12$

$$20 \times 5 = 100$$

 $20 \times 0.5 = 10$
 $20 \times 0.05 = 1$

Find families of facts from a known multiplication.

I know that $18 \times 4 = 72$.

This can help me work out:

$$1.8 \times 4 = ?$$

 $18 \times 0.4 = ?$
 $180 \times 0.4 = ?$
 $18 \times 0.04 = ?$

Use a place value grid to understand the effects of multiplying decimals.

	Н	Т	0	•	Tth	Hth
2 × 3			6	•		
0·2 × 3			0	•	6	
0·02 × 3						

Year 1 Division

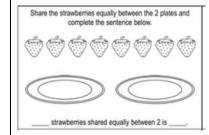
Pupils should be taught to:

• solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Division as sharing

Practice sharing into equal groups.





Complete a division calculation

8÷ 2 - 4

Year 2 Division

Pupils should be able to:

- recall and use division facts for the 2, 5 and 10
- calculate mathematical statements for multiplication and division
- write the division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Objectives and strategies	Concrete	Pictorial	Abstract	
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 + 2 = 4 Draw sharing model to support division and halving. Half of 10 is -	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?	

Division as grouping	Division by grouping- Group objects into their divisible factor.	Draw an array and use lines to split the array into groups to make multiplication and division sentences.					Find the inverse of multiplication and division sentences by creating four linking number sentences.		
		0	0		0	0	7 x 4 = 28 4 x 7 = 28		
			$\begin{array}{cccccccccccccccccccccccccccccccccccc$			28 ÷ 7 = 4 28 ÷ 4 = 7			
)			
Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.								
	Eg. 15 ÷ 3 = 5								
	5 x 3 = 15								
	15 ÷ 5 = 3								
	3 x 5 = 15								

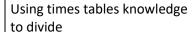
Division in quarters Dividing objects into 4 groups and counting the fraction of the amount.	2/4 of 20 = 10
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Division in thirds	Dividing objects into 3 groups and counting the fraction of the amount.	2 of 15=10 	¾ of 15 = 10
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Year 3 Division

Pupils should be taught to:

- recall and use division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects



Use knowledge of known times-tables to calculate divisions.



24 divided into groups of 8. There are 3 groups of 8.

Use knowledge of known times-tables to calculate divisions.



48 divided into groups of 4. There are 12 groups.

 $4 \times 12 = 48$ $48 \div 4 = 12$ Use knowledge of known times-tables to calculate divisions.

I need to work out 30 shared between 5.

I know that $6 \times 5 = 30$ so I know that $30 \div 5 = 6$.

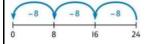
A bar model may represent the relationship between sharing and grouping.

			2	4		
F	4	4	4	4	4	4

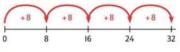
24 ÷ 4 = 6

 $24 \div 6 = 4$

Children understand how division is related to both repeated subtraction and repeated addition.

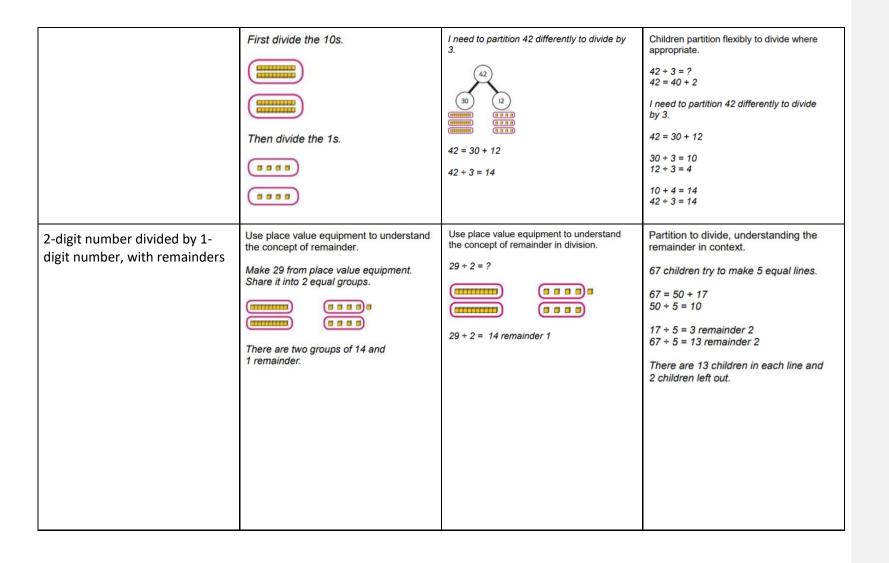


 $24 \div 8 = 3$



 $32 \div 8 = 4$

Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further. There are 13 sticks in total. There are 3 groups of 4, with 1 remainder.	Use images to explain remainders. 22 ÷ 5 = 4 remainder 2	Understand that the remainder is what cannot be shared equally from a set. $22 \div 5 = ?$ $3 \times 5 = 15$ $4 \times 5 = 20$ $5 \times 5 = 25 \dots$ this is larger than 22 So, $22 \div 5 = 4$ remainder 2
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. Make 6 ones divided by 3. Now make 6 tens divided by 3. What is the same? What is different?	Divide multiples of 10 by unitising. 12 tens shared into 3 equal groups. 4 tens in each group.	Divide multiples of 10 by a single digit using known times-tables. 180 + 3 = ? 180 is 18 tens. 18 divided by 3 is 6. 18 tens divided by 3 is 6 tens. 18 + 3 = 6 180 + 3 = 60
2-digit number divided by 1-digit number, no remainders	Children explore dividing 2-digit numbers by using place value equipment. 48 ÷ 2 = ?	Children explore which partitions support particular divisions.	Children partition a number into 10s and 1s to divide where appropriate. 68 60 + 2 = 30 8 + 2 = 4 30 + 4 = 34 68 + 2 = 34



Year 4 Division

Pupils should be taught to:

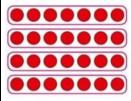
- recall division facts for multiplication tables up to 12 x 12
- use place value, known and derived facts to divide mentally
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Understanding the relationship between multiplication and division, including times-tables Use objects to explore families of multiplication and division facts.



4 × 6 = 24 24 is 6 groups of 4. 24 is 4 groups of 6.

24 divided by 6 is 4. 24 divided by 4 is 6. Represent divisions using an array.



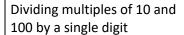
 $28 \div 7 = 4$

Understand families of related multiplication and division facts.

I know that $5 \times 7 = 35$

so I know all these facts:

 $5 \times 7 = 35$ $7 \times 5 = 35$ $35 = 5 \times 7$ $35 = 7 \times 5$ 35 + 5 = 7 35 + 7 = 5 7 = 35 + 55 = 35 + 7



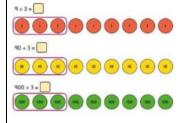
Use place value equipment to understand how to use unitising to divide.



8 ones divided into 2 equal groups 4 ones in each group

8 tens divided into 2 equal groups 4 tens in each group

8 hundreds divided into 2 equal groups 4 hundreds in each group Represent divisions using place value equipment.



 $9 \div 3 = 3$

9 tens divided by 3 is 3 tens. 9 hundreds divided by 3 is 3 hundreds. Use known facts to divide 10s and 100s by a single digit.

 $15 \div 3 = 5$

 $150 \div 3 = 50$

1500 ÷ 3 = 500

Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s

Partition into 10s and 1s to divide where appropriate.

$$39 \div 3 = ?$$



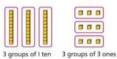
39 = 30 + 9

$$30 \div 3 = 10$$

 $9 \div 3 = 3$
 $39 \div 3 = 13$

Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate.

$$39 \div 3 = ?$$





$$39 = 30 + 9$$

$$30 \div 3 = 10$$

 $9 \div 3 = 3$

$$39 \div 3 = 13$$

Partition into 100s, 10s and 1s using a partwhole model to divide where appropriate.



$$100 \div 2 = 50$$

$$40 \div 2 = 20$$

$$6 \div 2 = 3$$

$$50 + 20 + 3 = 73$$

$$142 \div 2 = 73$$

6. TO + O Formal method with chunking



The chunking method is introduced but only with a single digit divisor. The number of groups should be recorded alongside on the right with the answer written on top of the bus stop.

TO ÷ O (with remainders)



The same layout is then used again but with remainders.

Year 5 Division

Pupils should be taught to:

- divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10, 100 and 1,000
- solve problems involving division, including using their knowledge of factors and multiples squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates

Understanding factors and prime numbers

Use equipment to explore the factors of a given number.

24 ÷ 3 = 8

24 ÷ 8 = 3 8 and 3 are factors of 24 because they divide 24 exactly.

24 + 5 = 4 remainder 4.

5 is not a factor of 24 because there is a remainder.

Understand that prime numbers are numbers with exactly two factors.

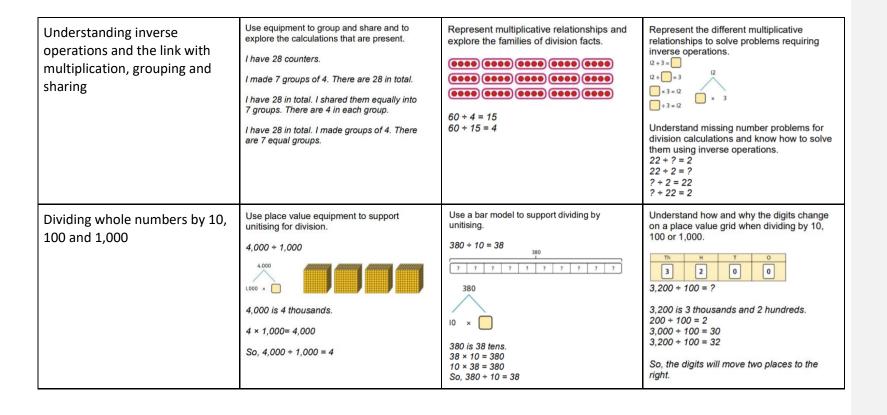
 $13 \div 1 = 13$ $13 \div 2 = 6 r 1$ $13 \div 4 = 4 r 1$

1 and 13 are the only factors of 13. 13 is a prime number. Understand how to recognise prime and composite numbers.

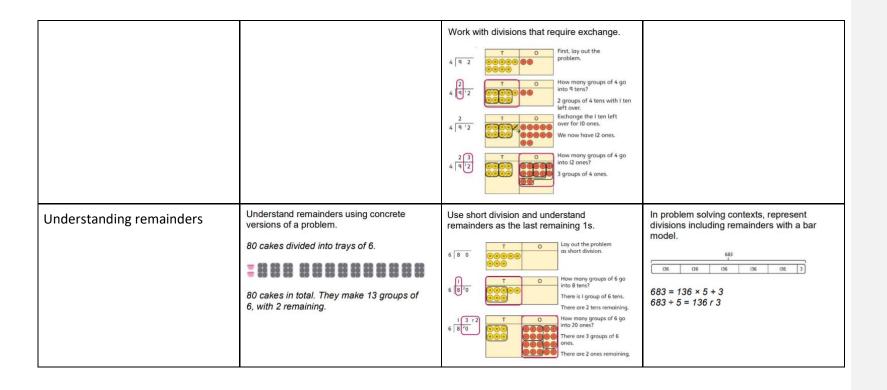
I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.

I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.

I know that 1 is not a prime number, as it has only 1 factor.



Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising. 15 ones put into groups of 3 ones. There are 5 groups. 15 tens put into groups of 3 tens. There are 5 groups. 15 tens put into groups of 3 tens. There are 5 groups.	Represent related facts with place value equipment when dividing by unitising. 180 is 18 tens. 18 tens divided into groups of 3 tens. There are 6 groups. 180 ÷ 30 = 6 11 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. $3,000+5=600$ $3,000+50=60$ $3,000+500=6$ $5\times600=3,000$ $50\times60=3,000$ $50\times60=3,000$
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. 268 ÷ 2 = ? There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones. 264 ÷ 2 = 134	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting. To o o o o o o o o o o o o o o o o o	Use short division for up to 4-digit numbers divided by a single digit. $ \begin{array}{cccccccccccccccccccccccccccccccccc$



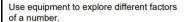
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. 2 ones are 20 tenths. 20 tenths divided by 10 is 2 tenths.	Represent division using exchange on a place value grid. The Hith Control Hith Con	Understand the movement of digits on a place value grid. O • Tth Hth Thth 0 • 8 • 5 0 • 90 • 18 • 3 0 • 85 ÷ 10 = 0.085 O • Tth Hth Thth 8 • 5 0 • 0 • 8 • 5 8 • 5 + 100 = 0.085
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. 1 whole shared between 3 people. Each person receives one-third.	Use a bar model and other fraction representations to show the link between fractions and division. $I \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$

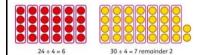
Year 6 Division

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations

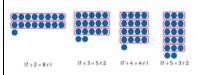






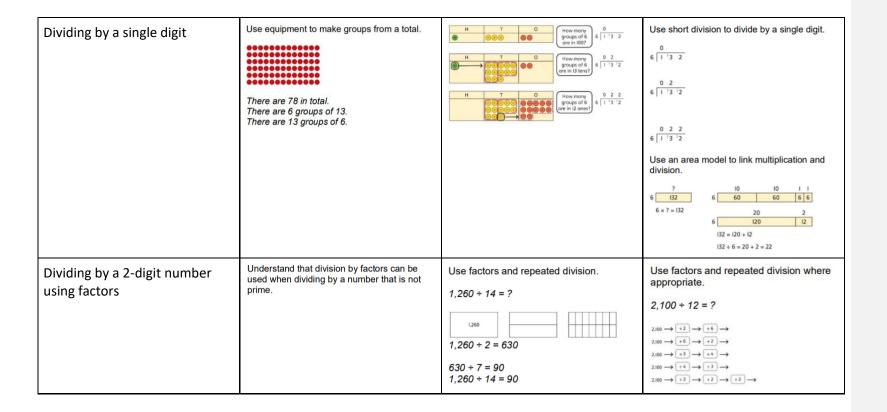
4 is a factor of 24 but is not a factor of 30.

Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.



Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.





Dividing by a 2-digit number using long division

Use equipment to build numbers from

182 divided into groups of 13. There are 14 groups.

Use an area model alongside written division to model the process.

377 ÷ 13 = ?



$$377 \div 13 = 29$$

Use long division where factors are not useful (for example, when dividing by a 2-digit prime number).

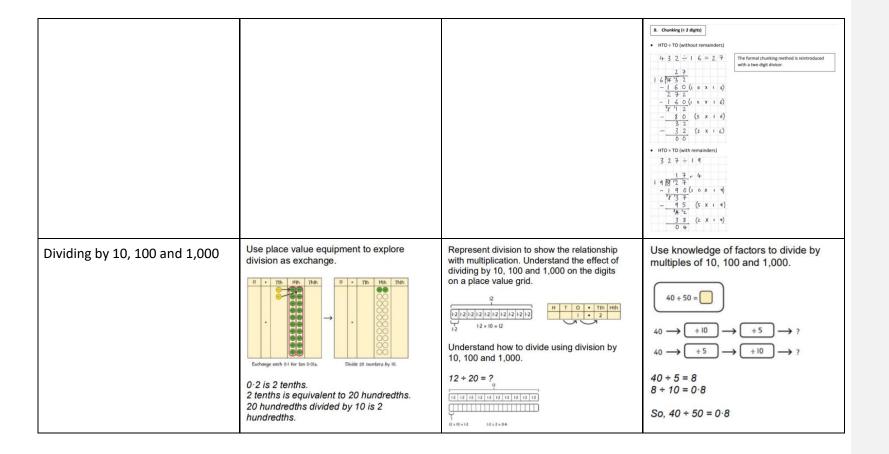
Write the required multiples to support the division process.



$$377 \div 13 = 29$$

A slightly different layout may be used, with the division completed above rather than at the side.

Divisions with a remainder explored in problem-solving contexts.

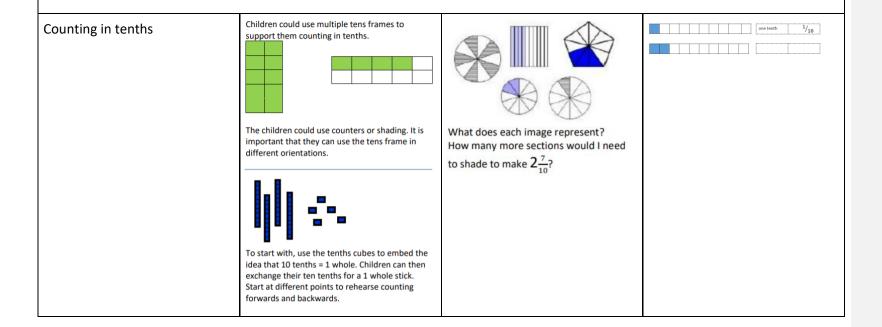


Use place value equipment to explore division of decimals. Dividing decimals Use a bar model to represent divisions. Use short division to divide decimals with up to 2 decimal places. 8 4 · 2 4 ? ? ? $4 \times 2 = 8$ $8 \div 4 = 2$ 8 4 · 42 4 8 tenths divided into 4 groups. 2 tenths in each group. So, $4 \times 0.2 = 0.8$ $0.8 \div 4 = 0.2$ 0 · 5 8 4 · 42 24 0 · 5 3 8 4 · 42 24

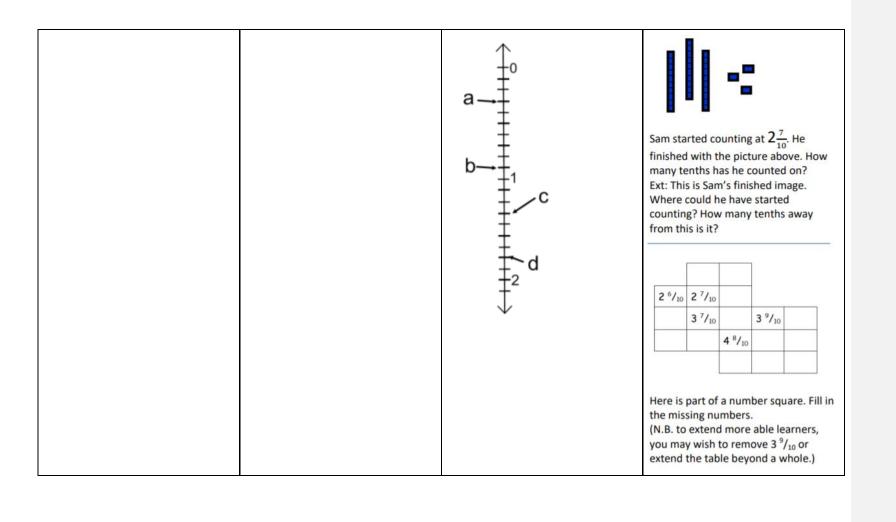
Year 3 Fractions

Pupils should be taught to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit number or quantities by 10.
- recognise, find and write fractions of a discrete set of objects, unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers, unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole for example, 5/7 + 1/7 = 6/7
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above.

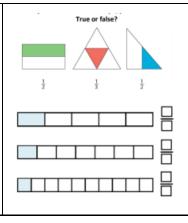


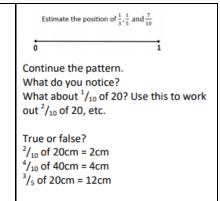
Commented [1]: Sorry!!! CJ asked me to add in a fractions bit too: I
We can do it for just ks2?



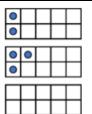
Identify fraction of shaded shape; position fractions on a number line; use fraction cards to show equivalence and compare fractions







Recognise that tenths arise from dividing an object into 10 equal parts and in dividing onedigit numbers or quantities by 10



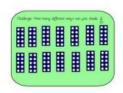
Use tens frames to represent tenths and count in tenths. Could also use a ten piece from numicon set with an object into the circles to represent the amount of tenths.

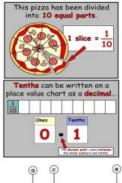
Using 10p coins with 10 adding up to £1 also links to the decimal place.

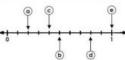




How many tenths of a whole pound do you have? 3/10 link to how it would be written as money £0.30.







Which tenth is represented by the letter a? B? C? D? E?

Eight tenths

7





What do you notice?

 $\frac{1}{10}$ of 10 = 1

 $\frac{2}{10}$ of 10 = 2 $\frac{3}{10}$ of 10 = 3

Continue the pattern.

What do you notice?

What about ¹/₁₀ of 20? Use this to work out $^{2}/_{10}$ of 20, etc.

 $^{1}/_{10}$ of 100 = 10

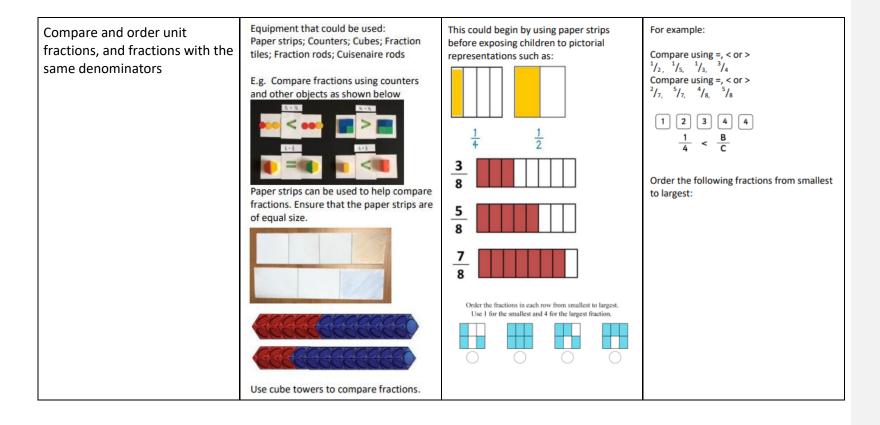
 $^{1}/_{100}$ of 100 = 1

 $^{2}/_{10}$ of 100 = 20

 $\frac{2}{100}$ of 100 = 2

How can you use this to work out $^{6}/_{10}$ of 200? ⁶/₁₀₀ of 200?





Add and subtract fractions with the same denominator within one whole



Provide pupils with a strawberry tart cut into eighths and an identically sized and cut blank copy.

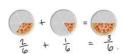
Collins Shanghai Y3 Unit 8.4



Determine that each part represents one eighth of the tart because the whole has been divided into eight equal parts. Get the children to cut out each part of the

pie and label them as $\frac{1}{8}$. Hold up one piece in each hand and elicit that this is $\frac{2}{8}$. Record the calculation:

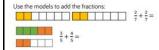
 $\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$. Relate the common denominators to the number of equal pieces of the tart, and then discuss how by adding two of them together they get $\frac{2}{8}$. Ask what would happen if one more eighth was added to the new strawberry tart. Stick another eighth on to get $\frac{3}{8}$. Continue this process. Put the final piece on and remind the children that $\frac{8}{2}$ is the same as one whole (strawberry tart).



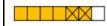


When using numicon, the base piece represents the denominator and the top pieces represent the numerator. Cubes or pegs could also be used to represent the numerators.

Cube towers can be used:



Similar resources can also be used to demonstrate subtraction of fractions.



Count up and down in fraction amounts on a number line.

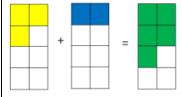


Twinkl Count up in fraction amounts using paper cards.



1 See





Make sure the numerators are the same, then add the denominators.

e.g.
$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$



Year 4 Fractions

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to 1/4, 1/2, 3/4
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

Counting up and down in hundredths.

Use 100 bead strings to rehearse counting forwards and backwards. Use different starting points and draw discussion into the fact that $^{1}/_{10}$ = $^{10}/_{100}$.



To start with, use the hundredths counters to embed the idea that 10 hundredths = 1 tenth. Children can then exchange their ten hundredths for a tenth counter. Start at different points to rehearse counting forwards and backwards.

As for counting in tenths
Use dienes units blocks to represent
1/100
Use pennies to recognise 1/100 of a
pound etc

Decimal Number Chart 0.01-1

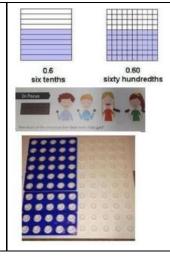
0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.3
0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4
0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.6
0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.7
0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.8
0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.9
0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1
	0.12 0.22 0.32 0.42 0.52 0.62 0.72	0.12 0.13 0.22 0.23 0.32 0.33 0.42 0.43 0.52 0.53 0.62 0.63 0.72 0.73 0.82 0.83	0.12 0.13 0.14 0.22 0.23 0.24 0.32 0.33 0.34 0.42 0.43 0.44 0.52 0.53 0.54 0.62 0.63 0.64 0.72 0.73 0.74 0.82 0.83 0.84	0.12 0.13 0.14 0.15 0.22 0.23 0.24 0.25 0.32 0.33 0.34 0.35 0.42 0.43 0.44 0.45 0.52 0.53 0.54 0.55 0.62 0.63 0.64 0.65 0.72 0.73 0.74 0.75 0.82 0.83 0.84 0.85	0.12 0.13 0.14 0.15 0.16 0.22 0.23 0.24 0.25 0.26 0.32 0.33 0.34 0.35 0.36 0.42 0.43 0.44 0.45 0.46 0.52 0.53 0.54 0.55 0.56 0.62 0.63 0.64 0.65 0.66 0.72 0.73 0.74 0.75 0.76 0.82 0.83 0.84 0.85 0.86	0.12 0.13 0.14 0.15 0.16 0.17 0.22 0.23 0.24 0.25 0.26 0.27 0.32 0.33 0.34 0.35 0.36 0.37 0.42 0.43 0.44 0.45 0.46 0.47 0.52 0.53 0.54 0.55 0.56 0.57 0.62 0.63 0.64 0.65 0.66 0.67 0.72 0.73 0.74 0.75 0.76 0.77 0.82 0.83 0.84 0.85 0.86 0.87	0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.42 0.43 0.44 0.45 0.46 0.47 0.48 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.82 0.83 0.84 0.85 0.86 0.87 0.88	0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.39 0.42 0.43 0.44 0.45 0.46 0.47 0.48 0.49 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.79 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99

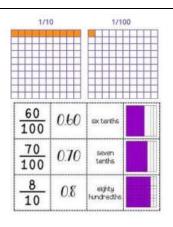
As for counting in tenths
9.8 is equivalent to 980 hundredths,
9.82 is equivalent to ? hundredths.
How many hundredths are there in
10.0?

Using a partially filled in hundredths square, ask how you know where e.g. 2.76 would go.

2.5	2.5			
2	3			
				2.6
				7
2.8).	2.8	2.8	
2		5	6	
			2.9	
			6	

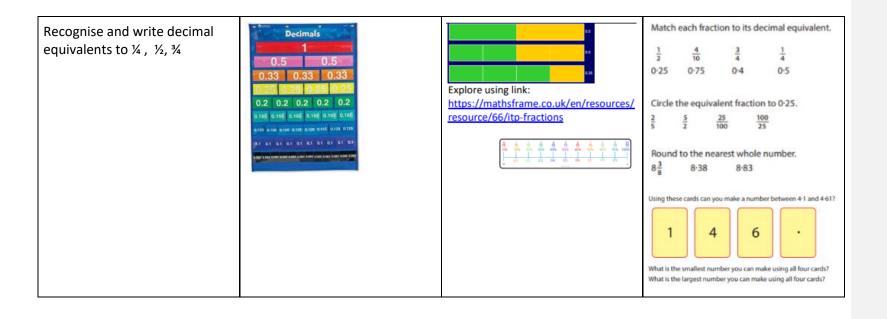
Recognise and write decimal equivalents of any number of tenths or hundredths





What do you notice?

One tenth of £41 One hundredth of £41 One thousandth of £41 Continue the pattern. What do you notice? 0.085 + 0.015 = 0.1 0.075 + 0.025 = 0.1 0.065 + 0.035 = 0.1Continue the pattern for the next five number sentences



Add and subtract fractions with the same denominator including bridging over whole numbers e.g. 7/9 + 4/9 = 11/9 or 1 whole and 2/9

As before, use cubes and numicon to create the fractions:

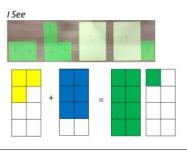




Count up and down in fraction amounts on a number line.



Twinkl
Count up in fraction amounts using paper cards.



Make sure the numerators are the same, then add the denominators. If your answer is an improper fraction, convert it to a mixed number if the problem requires it.

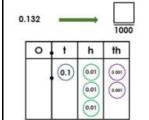
e.g.
$$\frac{7}{9} + \frac{4}{9} = \frac{11}{9}$$
 or $1\frac{2}{9}$

Year 5 Fractions

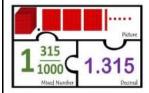
Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same numbers
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 5 2 + 5 4 = 5 6 = 1 5 1]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams read and write decimal numbers as fractions [for example, 0.71 = 100 71]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the percent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25.

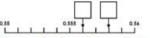
Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents



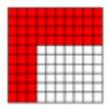
Use place value counters and grid to represent decimal numbers up to three decimal points and convert to fractions



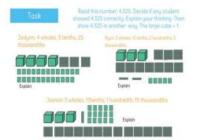
Using base ten to physically represent decimal numbers.



Use number lines to represent thousandths as the steps between hundredths.



Using a tens frame, 100 square, or thousands grid to represent tenths, hundredths and thousandths



One tenth of £41 One hundredth of £41 One thousandth of £41 Continue the pattern What do you notice?

0.085 + 0.015 = 0.1 0.075 + 0.025 = 0.10.065 + 0.035 = 0.1

Continue the pattern for the next five number sentences.

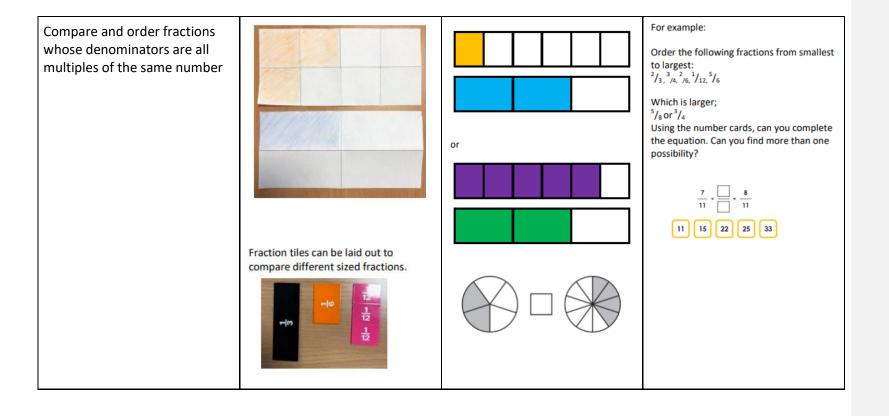
One thousandth of my money is 31p. How much do I have?

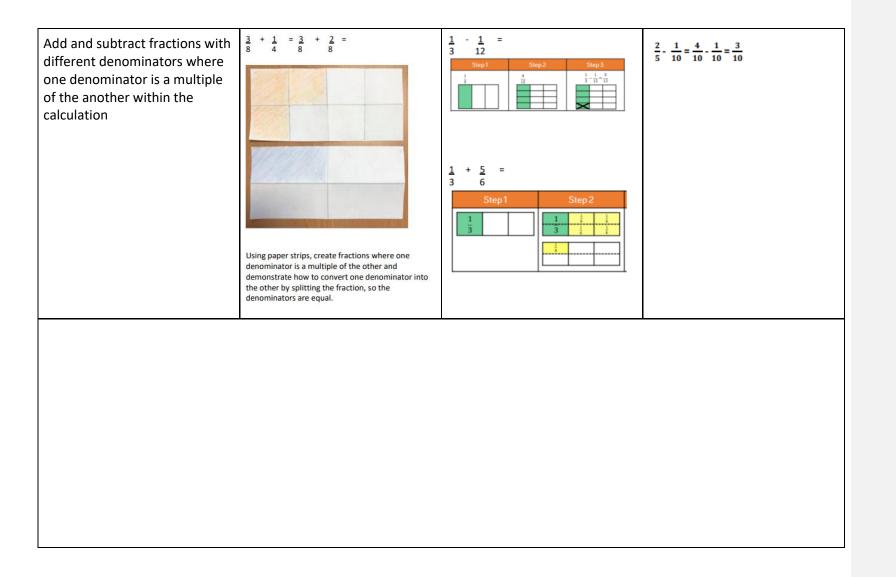
True or false?
0.1 of a kilometre is 1m.
0.2 of 2 kilometres is 2m.
0.3 of 3 kilometres is 3m
0.25 of 3m is 500cm.
²/₅ of £2 is 20p

True or false?

25% of 23km is longer than 0.2 of 20km.

Convince me.

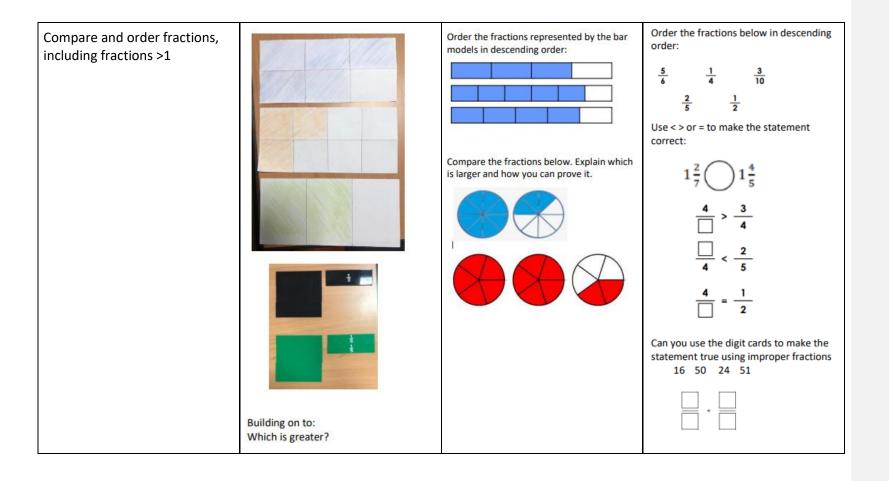


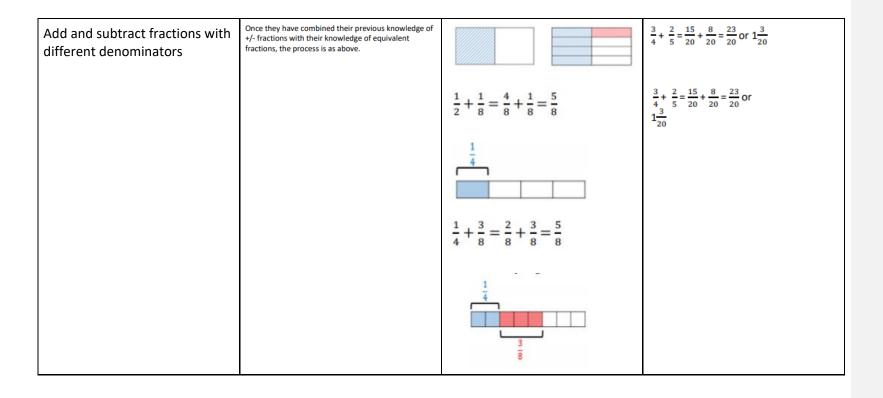


Year 6 Fractions

Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $4.1 \times 2.1 = 8.1$]
- divide proper fractions by whole numbers [for example, $31 \div 2 = 61$]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

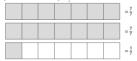




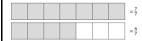
Add and subtract fractions, including mixed numbers and improper fractions where the no bridging is required

Addition

Provide pupils with strips split into sevenths. Elicit that $\frac{15}{\pi}$ is the same $2\frac{1}{\pi}$



Elicit that $\frac{11}{7}$ is the same $1\frac{4}{7}$



Question: How many sevenths do we have altogether? Twenty six sevenths.

Reminder: when adding/subtracting two fractions, the denominator stays the same – even when it is an improper fraction.

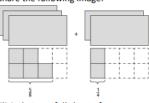
Elicit that $\frac{26}{7}$ is equivalent to $3\frac{5}{7}$

Use a similar example to model the same for subtraction.

Oscar was running a race. He ran $3^{5}/_{8}$ of a kilometre in the first 15 minutes.

He ran 2 ¹/₄ of a kilometre in the second 15 minutes.

How far had he run in total after 30 minutes? Share the following image:



Elicit that one full sheet of paper represents 1km. Agree that he has run five full kilometres because there are five full sheets.

Ask: Oscar ran further than 5 km. How much further? Agree that Oscar has also ran $\frac{5}{8}$ km

and $\frac{1}{4}$ km. Establish that these are related fractions because one denominator is a multiple of the other.

Share the following then get pupils to discuss what is happening:

$$3\frac{5}{8} + 2\frac{1}{4} = 3\frac{5}{8} + 2\frac{2}{8}$$

$$= (3 + \frac{5}{8}) + (2 + \frac{2}{8})$$

$$= (3 + 2) + (\frac{5}{8} + \frac{2}{8})$$

$$= (5 + \frac{7}{8})$$

$$= 5\frac{7}{8}$$

Collins Shanghai Y6 Unit 6.7

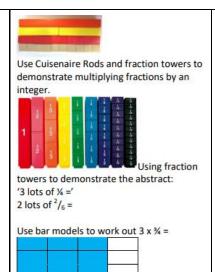
Use a similar example to model the same for subtraction.

$$3\frac{3}{4} + \frac{1}{5} = 3\frac{19}{20}$$

$$11\frac{3}{4} - 7\frac{1}{6} = 4\frac{7}{12}$$

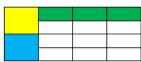
Addition Add and subtract fractions, For example Example: $3\frac{5}{7} + 2\frac{4}{7}$ including mixed numbers and Use the same method as previously with the strips, but get the children to cut up the fraction elements improper fractions where into the component sevenths. This would leave 5 wholes (5 strips of $\frac{7}{7}$) and nine $\frac{1}{7}$ bridging is required parts. Elicit that nine $\frac{1}{7}$ parts is the same as $\frac{9}{7}$ or $1\frac{2}{7}$ parts. Combine 5 with $1\frac{2}{7}$ to get the answer of $6\frac{2}{7}$ Subtraction Example: $4^{\frac{2}{3}} - 2^{\frac{5}{3}}$ Use the same method as previously with the strips, but get the children to cut up the fraction elements into the component sevenths. This would leave 2 wholes (2 strips of $\frac{7}{7}$) but then they would realise that they cannot take $\frac{5}{2}$ away from $\frac{2}{3}$. Discuss what could be done next. Elicit that they could cut one of their two wholes into $\frac{7}{2}$. Going back to their original $4\frac{2}{3}$ they would the repartition that into $3\frac{9}{7}$. With 3 $\frac{9}{7}$ in strips, they could remove the 2 wholes then the $\frac{4}{7}$ leaving them with $1\frac{5}{7}$.

Multiply simple pairs of proper fractions, writing the answer in its simplest form Multiply fractions by whole numbers

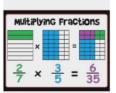


Use a number line to work it out:

Use a diagram to represent multiplying fractions. Build an array (as used when multiplying whole numbers For example $\frac{1}{2}$ x $\frac{2}{4}$



Draw a bar and shade $\frac{1}{4}$ Draw an adjoining column and shade $\frac{2}{4}$. The shaded cells represent the total. $\frac{2}{16}$ or $\frac{1}{8}$

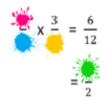


Solve:

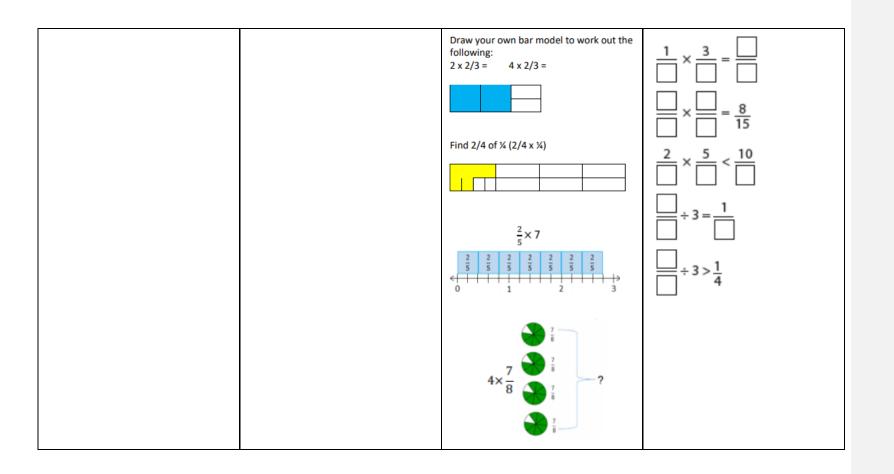
$$\frac{2}{2} \times - = \frac{6}{20} = -$$

$$- \times \frac{10}{5} = \frac{10}{60} = -$$

How many ways can you answer the following?



In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true.



Divide proper fraction by whole numbers.

Using paper strips is a good way to demonstrate what happens to a fraction when it is divided by a whole integer i.e. 2

Fold the paper in half:

What happens when I divide 1/2 by 2? By folding each half into two parts, it becomes clear that quarters have been formed. Lee has $\frac{2}{5}$ of a chocolate bar. He shares it with his friend. How much chocolate do they get each?



Use the diagrams to help you calculate:





Calculate the following and use the diagram to help you.

$$\frac{1}{8} \div 4 =$$



Solve:

$$6 \div -= 9$$

$$--$$
 ÷ $\frac{2}{5}$ = 10

$$-\div$$
___= 5 $\frac{5}{6}$

Harjoht's Mum ordered pizza for the whole family. Harjoht ate ¼ of the pizza. His Mum, brother and sister ate the rest of the pizza. What fraction of the pizza did they get each?

Molly's Mum ordered pizza for the whole family. Molly ate ¼ of the pizza. Six members of the family eat the remaining pizza. What fraction of the pizza did they get each?

Vocabulary

Addition & Subtraction

Years 3 and 4: add, addition, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? how many more is...? how many more is....? how much more is...? -, subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? how many fewer is... than...? how much less is...? difference between, half, halve, how many more/fewer is... than...? how much more/less is...? Is equal to, is the same as, tens boundary, hundreds boundary, inverse

Years 5 and 6: add, addition, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is... than...? how much more/less is...? Is equal to, sign, is the same as, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse

Multiplication & Division

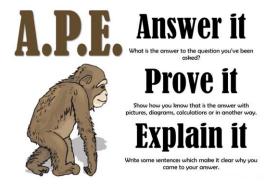
Year 3 and 4: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product, once, twice, three times... ten times...times as (big, long, wide... and so on), repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three each...group in pairs, threes... tens, equal groups of, divide, division, divided by, divided into, remainder, factor, quotient, divisible by, inverse

Years 5 and 6: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product once, twice, three times... ten times...times as (big, long, wide... and so on), repeated addition array, row, column, double, halve, share, share equally, one each, two each, three each...group in pairs, threes... tens, equal groups of, divide, division, divided by, divided into, dividend, divisor, remainder, factor, quotient, divisible by, inverse, fraction

Reasoning and Problem Solving

All children to be given the opportunity to reason and problem solve at least once a week.

Children should be using the Answer Prove it Explain it guidelines when reasoning.



See Reasoning Examples for further guidance.