The Poppy Academy Trust
Calculation Policy 2022


St. John's Nursery and Infant School Fair Field Junior School

EYFS - Year 6 Calculation Policy

This Calculation Policy has been produced in line with the 2014 National Curriculum for Mathematics to ensure consistency and progression in teaching throughout the school that is age appropriate. It aims to introduce children to the processes of calculation through concrete, pictorial and abstract activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases and learn to interpret and use the signs and symbols involved. This policy shows the natural progression that a child should make in their mathematical education. Children should not progress onto the advanced stages of formal written methods until they have a secure conceptual understanding.

## Intent

Maths is a journey and long-term goal, achieved through exploration, clarification, practice and application over time. At each stage of learning, children should be able to demonstrate a deep, conceptual understanding of the topic and be able to build on this over time.

Our overall aims for when children leave St. John's Nursery and Infants and Fair Field Junior School are:

- we ensure children feel confident to apply their skills to everyday life and understand that maths is all around us and intrinsic to a successful life
- develop a positive attitude to mathematics as a subject in which all children gain success and pleasure.
- have access to a high quality maths curriculum that is both challenging and enjoyable, and builds upon previous learning.
- be provided with a variety of mathematical opportunities, which will enable them to make relevant connections.
- ensuring children are confident mathematicians who are not afraid to take risks.
- children are confident when reasoning and make appropriate decisions, applying mathematical thinking in order to solve problems.
- develop an ability to express themselves fluently, to talk about the subject with assurance, using correct mathematical language and vocabulary.
- develop mathematical skills and knowledge and recall of basic number facts and the four operations
- be able to use this knowledge and understanding to carry out calculations mentally
- make use of pictorial representations and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally.
They will do this by always asking themselves: Can I do this in my head? Can I do this in my head using pictorial representations? Do I need to use a pencil and paper procedure of a formal written method?

Implementation

The school uses Herts For Learning 'ESSENTIAL Maths' as the scheme of work that provides continuity and progression ensuring appropriate pitch and coverage of the curriculum. We continue to modify to match with our school's approach and the needs of our pupils.

This policy is a statement of the aims, principles and strategies for teaching and learning of calculation strategies in Mathematics. It is designed to help teachers and staff at St. John's Nursery and Infant School and Fair Field Junior School ensure that calculation is taught consistently across the school and to aid them in helping children who may need extra support or challenges. This policy is also designed to help parents, carers and other family members support children's learning by providing an explanation of the methods used in our school.

The policy is set out according to the 4 operations, addition, subtraction, multiplication and division; and incorporates fractions for KS2. Within each specific area there is a progression of skills, knowledge and layout for written methods. The calculation strategies which will be used will reflect this ideology - moving from concrete to pictorial and then abstract recording leading to more formal written methods. The expectations are that teachers follow this to ensure consistency in approach and delivery at the appropriate stage. Mental methods and strategies will work in partnership with these methods. A variety of mental calculation methods will be taught and that recall of facts will be taught in school and tested regularly. The progression of mental methods and expectations will comply with the National Curriculum Statements 2014. It is important staff throughout the trust use the correct mathematical language and encourage pupils to do this also, which will help them develop confidence in their mathematical processes and reasoning. This will take place in class discussions as well as through oral and written feedback, next steps and target setting. The basis of our maths calculation policy is that written methods are complementary to mental methods and should not be seen as separate from them.

## The Importance of Mental Mathematics

While this policy focuses on written calculation in mathematics, we recognise the importance of mental strategies and known facts
that form the basis of all calculations. Pupils are provided with frequent opportunities to compare and evaluate different calculation strategies. This helps them develop an understanding that efficiency is personal and based on the numbers involved. Mental maths fluency underpins all effective written calculation approaches.

## Concrete, Pictorial and Abstract

Concrete - Manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways. All pupils should have frequent opportunities to develop their understanding of mathematical concepts through the appropriate use of concrete apparatus.

Concrete resources that may be found in classrooms will include:


These resources will vary depending on year group and individual needs. At home, pupils very well may not have access to these school resources; however, they are just a vehicle to support a pupil's understanding of a topic. Any objects can be used at home to replace counters, cubes etc.

Pictorial (including jottings) - The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives. When a child is working at the pictorial stage, it often provides rich opportunities for assessment of their depth of understanding.

Abstract - Written The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

Abstract - Spoken Learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using speaking frames.

## Impact

Pupils will leave us prepared for the next stage in their lives with:

- Pupils have an appreciation for the maths in everyday life
- Quick recall of facts and procedures
- The flexibility and fluidity to move between different contexts and representations of mathematics
- The ability to recognise relationships and make connections in mathematics
- Confidence and belief that they can achieve
- The knowledge that maths underpins most of our daily lives
- Skills and concepts that have been mastered
- Have a positive and inquisitive attitude to mathematics as an interesting and attractive subject in which all children gain success and pleasure.

A mathematical concept or skill has been mastered when a child can show it in multiple ways, using the mathematical language to explain their ideas, and can independently apply the concept to new problems in unfamiliar situations and this is the goal for our children. These will be assessed through: assessment, tracking, pupil progress meetings, performance management, moderation and standardisation.

## Early Years Foundation Stage

Children at the expected level of development will:

- Have a deep understanding of number to 10 , including the composition of each number
- Subitise (recognise quantities without counting) up to 5
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10 , including double facts.

| Addition | Subtraction | Multiplication | Division |
| :---: | :---: | :---: | :---: |
| Children are encouraged to gain a sense of the number system through the use of counting concrete objects. <br> 7 <br> ways and count all. <br> They understand addition as counting on and will count on in ones and twos using object s, cubes, bead string and number line. <br> They use concrete and pictorial representation to record their calculations. <br> When confident children may be able to pictorially represent their calculations using symbols and | Children are encouraged to gain a sense of the number system through the use of counting concrete objects and understand subtraction as counting out. <br> They begin to count back in ones and twos using objects, cubes, bead string and number line. <br> They may use concrete and pictorial representation to record their calculations. <br> They are encouraged to develop a mental picture of the number system in their heads to use for calculations. When confident children may be | Children use concrete objects to make and count equal groups of objects. <br> They will count on in twos using a bead string and number line. <br> They understand doubling as repeated addition. $4+4=8$ <br> They use concrete and pictorial representation to record their calculations. When confident | Children use concrete objects to count and share equally into 2 groups. <br> They count a set of objects and halve them by making two equal groups. <br> They understand sharing and halving as dividing by 2 . |


| numbers within a written | able to represent their calculations <br> using symbols and numbers within a <br> written calculation. | children may be able to represent <br> their calculations using symbols and <br> numbers within a written <br> calculation. | They will begin to use objects to <br> make groups of 2 from a given <br> amount. They use concrete and <br> pictorial representation to record <br> their calculations. |
| :--- | :--- | :--- | :--- |

## Year 1 Addition

Pupils should be able to:

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20 , including zero solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $7=-9$.

| Combining two parts to make a whole: part- whole model | Use part part whole model. <br> Use cubes to add two numbers together as a group or in a bar. |  | Part whole model drawing |
| :---: | :---: | :---: | :---: |
| Starting with the biggest number and counting on | Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. | $12+5=17$ <br> Start at the larger number on the number line and count on in ones or in one jump to find the answer. | Place the larger number in your head and count on the smaller number to find your answer. |



## Year 2 Addition

Pupils should be taught to:

- add numbers using concrete objects, pictorial representations, and mentally including:
- a two-digit number and ones a two-digit number
- tens two two-digit numbers
- adding three one-digit numbers
- solve problems with addition: using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition facts to 20 fluently, and derive and use related facts up to 100


| Add two 2digit numbers (without regrouping) |  <br> Using base-ten, numicon and place value counters to support adding two 2-digit numbers without regrouping. | Tens Ones <br> $\\|\\|$ $\cdots$ <br> $+\\| \\|$ $\cdots \cdots$ <br> $\\|\\|\\|\\|$ $\cdots \cdots$ <br>  $\cdots$ <br> When confident they will begin to draw the expanded written method. <br> We provide lots of strategies for adding two digit numbers for example number lines - | Adding tens and ones - $\begin{array}{r} 403 \\ +\quad 30 \quad 5 \\ \hline 708 \\ \hline \end{array}$ $43+35=78$ <br> When confident they will begin to use the expanded written method to add two digit numbers. |
| :---: | :---: | :---: | :---: |



| Add three 1- <br> digit <br> numbers. | ( |
| :--- | :--- | :--- | :--- | :--- |

## Year 3 - Addition

## Pupils should be taught to:

- add numbers mentally, including:
- a three-digit number and 1s
- a three-digit number and 10 s
- a three-digit number and 100s
- add numbers with up to 3 digits, using formal written methods of columnar addition
- solve problems, including missing number problems, using number facts, place value, and more complex addition.

Bar models to be used to support decision making and where the missing numbers fit in our calculations.






## Year 4 - Addition

Pupils should be taught to:

- Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition two-step problems in contexts, deciding which operations and methods to use and why

Bar models to be used to support decision making and where the missing numbers fit in our calculations.

| Adding a multiple of 1000 or 100 to a 4 digit number |  | $\lceil 1700+1400$ |  | $1700+1400$ |
| :---: | :---: | :---: | :---: | :---: |
| Mental calculations (rounding, doubling, using number bonds, adding near doubles) |  |  | $\underbrace{1376+1598}_{1374+2}$ | $1374+1600$ |
| Add numbers to one decimal place | Add numbers to one decimal place $1.8+0.7$ | $1.8+0.7$ <br> :8888 88000 <br> :88:8 <br> 00000 <br> $888^{\circ \circ}$ $\square$ $\mathbf{8 8 8 8}$ $\square$ | $\overbrace{0.2}^{0.7}$ |  |



## Addition Year 5

Pupils should be taught to:

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Add numbers mentally with increasingly large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Bar models to be used to support decision making and where the missing numbers fit in our calculations.

| Mental <br> calculations <br> (rounding, <br> doubling, using <br> number bonds, <br> adding near <br> doubles) |  | Examples |
| :--- | :--- | :--- | :--- |
|  |  | $1445+\mathbf{2 9 9 9}$ |
|  |  | $1445+3000-1$ |
|  |  | $\mathbf{1 2 9 9 + 1 2 9 9}$ |
| Double $1300-2$ |  |  |



| Adding Tenths | Link measure with addition of decimals. <br> Two lengths of fencing are 0.6 m and 0.2 m. <br> How long are they when added together? | Use a bar model with a number line to add tenths. $0.6+0.2=0.8$ <br> 6 tenths +2 tenths $=8$ tenths | Understand the link with adding fractions. $\begin{aligned} & \frac{6}{10}+\frac{2}{10}=\frac{8}{10} \\ & 6 \text { tenths }+2 \text { tenths }=8 \text { tenths } \\ & 0.6+0.2=0.8 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Adding decimals using column addition | Use place value equipment to represent additions. <br> Show $0.23+0.45$ using place value counters. | Use place value equipment on a place value grid to represent additions. <br> Represent exchange where necessary. <br> Include examples where the numbers of decimal places are different. $$ | Add using a column method, ensuring that children understand the link with place value. $\begin{array}{r} \mathrm{O} \cdot \text { Tth Hth } \\ \hline 0 \cdot 2 \\ +0 \cdot 4 \\ \hline 0 \cdot 4 \\ \hline 0 \cdot 6 \\ \hline \end{array}$ <br> Include exchange where required, alongside an understanding of place value. $$ <br> Include additions where the numbers of decimal places are different. $3.4+0.65=?$ $\begin{array}{r} 0 \cdot \text { Tth Hth } \\ \hline 3 \cdot 40 \\ +0 \cdot 6 \quad 5 \end{array}$ $\qquad$ |

## Addition Year 6

Pupils should be taught to:

- perform mental calculations, using increasingly large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition multi-step problems in contexts, deciding which methods to use and why solve problems involving addition

Bar models to be used to support decision making and where the missing numbers fit in our calculations


Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.


Use bar model and number line representations to model addition in problem-solving and measure contexts


Use column addition where mental methods are not efficient. Recognise common errors with column addition.
$32,145+4,302=?$

$$
\begin{aligned}
& \begin{array}{ccccc}
\text { TTh Th } & \text { H } & \text { T } & \text { O } \\
\hline 3 & 2 & 1 & 4 & 5
\end{array} \\
& \begin{array}{r}
4302 \\
\hline 6447
\end{array}
\end{aligned}
$$

Which method has been completed accurately?

What mistake has been made?
Column methods are also used for decimal additions where mental methods are not efficient.

$$
\begin{array}{rcccc}
\mathrm{H} & \mathrm{~T} & \mathrm{O} \cdot \mathrm{Tth} & \mathrm{Hth} \\
\hline 1 & 4 & 0 & \cdot 0 & \mathrm{q} \\
+ & 4 & \mathrm{q} \cdot & 8 & \mathrm{q} \\
\hline
\end{array}
$$

| Selecting mental methods for larger numbers where appropriate | Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods. $2,411,301+500,000=?$ <br> This would be 5 more counters in the HTh place. <br> So, the total is $2,911,301$. $2,411,301+500,000=2,911,301$ | Use a bar model to support thinking in addition problems. <br> I added 100 thousands then subtracted 1 thousand. <br> 257 thousands +100 thousands $=357$ thousands $\begin{aligned} & 257,000+100,000=357,000 \\ & 357,000-1,000=356,000 \end{aligned}$ <br> So, $257,000+99,000=356,000$ | Use place value and unitising to support mental calculations with larger numbers. $\begin{aligned} & 195,000+6,000=? \\ & 195+5+1=201 \end{aligned}$ <br> 195 thousands +6 thousands $=201$ thousands <br> So, $195,000+6,000=201,000$ |
| :---: | :---: | :---: | :---: |
| Understanding order of operations in calculations | Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. | Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. | Understand the correct order of operations in calculations without brackets. <br> Understand how brackets affect the order of operations in a calculation. $\begin{aligned} & 4+6 \times 16 \\ & 4+96=100 \\ & (4+6) \times 16 \\ & 10 \times 16=160 \end{aligned}$ |

## Year 1 Subtraction

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction ( - ) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20 , including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7=-$ 9.

| Taking away ones | Use physical objects, counters, cubes etc. to show how objects can be taken away. $6-2=4$ | Cross out drawn objects to show what has been taken away. $15-3=12$ | Show subtraction number sentence. $18-3=15$ $8-2=6$ |
| :---: | :---: | :---: | :---: |
| Counting back | Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. <br> Use counters and move them away from the group as you take them away counting backwards as you go. | Count back on a number line or number track. <br> Start at the bigger number and count back the smaller number showing the jumps on the number line. <br> This can progress all the way to counting back using two 2 digit numbers. | Put 13 in your head, count back 4. What number are you at? Use your fingers to help. $13-4=$ |


| Find the difference | Compare amounts and objects to find the difference. <br> Use cubes to build towers or make bars to find the difference. <br> Use basic bar models with items to find the difference | Count on to fo ind the difference. <br> Draw bars to find the difference between 2 numbers. <br> Comparison Bar Models Lisc is 13 years otd. Her sister is 22 years old. thern. $\begin{aligned} & \text { Liso } \\ & \text { Siater } \end{aligned}$ $\square$ <br> 22 | Hannah has 23 sandwiches, Helen has 15 sandwiches. <br> Find the difference between the number of sandwiches. $23-15=$ |
| :---: | :---: | :---: | :---: |
| Part Part Whole Model | Make 14 on the ten frame. <br> Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. <br> $140-$ | $13-7=$ $\qquad$ <br> Start at 13. Take away 3 to reach 10 . Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. | How many do we take off to reach the next 10 ? How many do we have left to take off? $16-8=$ |

## Year 2 Subtraction

Pupils should be able to:

- solve problems with subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers

| Objectives and strategies | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Regroup a ten into ten ones and subtract. | 1 ten <br> 10 ones | $\begin{aligned} & 13-5=8 \\ & 100 \\ & 00000 \\ & 000 \times x \\ & x \times x \end{aligned}$ | 13-5 = 8 |



| Subtract two digit numbers with regrouping. | Make tens and ones and then regroup one ten for ten ones to be able to subtract the number. | Children will assess if they need to use regrouping to solve the two digit subtraction question. Draw the method they need to use. | When confident, children will begin to use the expanded written method to subtract tens and ones. $\begin{array}{r} 6013 \\ 70+3 \\ -\quad 40+6 \\ \hline 20+7 \\ \hline \end{array}$ $73-46=27$ |
| :---: | :---: | :---: | :---: |

## Subtraction Year 3

Pupils should be taught to:

- subtract numbers mentally, including:
- a three-digit number and 1s
- a three-digit number and 10s
- a three-digit number and 100s
- subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction

Children should also be taught to calculate the difference when two numbers are close in range e.g. $114-98$, counting on $98+2=100$ then $100+14=114$, therefore the difference is 16 at all stages.

Bar models to be used to support decision making and where the missing numbers fit in our calculations.




| Column method without regrouping | Use Base 10 to make the bigger number then take the smaller number away. <br> Show how you partition numbers to subtract. Again make the larger number first. |  | $\begin{gathered} 47-24=23 \\ -40+7 \\ -20+4 \\ 20+3 \\ \hline \end{gathered}$ <br> This will lead to a clear <br> written column subtraction. |
| :---: | :---: | :---: | :---: |
| Column method with regrouping | Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. <br> Make the larger number with the place value counters |  | $\begin{gathered} 728-582=146 \\ \prime \prime \\ 4 \\ 7 \\ 7 \\ 5 \\ 5 \\ 5 \\ 1 \end{gathered}$ |



## Subtraction Year 4

Pupils should be taught to:

- subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why

Bar models to be used to support decision making and where the missing numbers fit in our calculations.


| Subtract with numbers up to four digits, including exchanging | Understand why exchange of a 1,000 for 100 s, a 100 for 10 s, or a 10 for 1 s may be necessary. <br> $\rightarrow$ ggaga | Represent place value equipment on a place value grid to subtract, including exchanges where needed. | Use column subtraction, with understanding of the place value of any exchange required. |
| :---: | :---: | :---: | :---: |


| Column subtraction with exchange across more than one column | Understand why two exchanges may be necessary. $2,502-243=?$ <br> I need to exchange a 10 for some 1 s, but there are not any 10 s here. $\rightarrow$ 品品 | Make colum holde 2,502 $\square$ | hanges across more than one where there is a zero as a place $243=?$  | Make exchanges across more than one column where there is a zero as a place holder. <br> $2,502-243=$ ? |
| :---: | :---: | :---: | :---: | :---: |



## Subtraction Year 5

Pupils should be taught to:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- solve subtraction multi-step problems in contexts, deciding which methods to use and why




## Subtraction Year 6

Pupils should be taught to:

- perform mental calculations, including with increasingly large number
- use their knowledge of the order of operations to carry out calculation involving the 4 operations
- solve subtraction multi-step problems in contexts, deciding which methods to use and why
- solve problems using subtraction

| Comparing and selecting | Use counters on a place value grid to represent subtractions of larger numbers. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Th | H | T | 0 |
|  | - ${ }^{\circ}$ | O2202 | $\begin{aligned} & \odot \odot \odot \odot Q \\ & Q Q \end{aligned}$ |  |

## Compare subtraction methods alongside

 place value representations.

Use a bar model to represent calculations, including 'find the difference' with two bars as comparison.


Compare and select methods use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy.


Use column subtraction for decimal problems, including in the context of measure.


| Subtracting mentally with larger numbers |  | Use a bar model to show how unitising can support mental calculations. $950,000-150,000$ <br> That is 950 thousands -150 thousands $\square$ <br> 800 <br> So, the difference is 800 thousands. $950,000-150,000=800,000$ | Subtract efficiently from powers of 10. $10,000-500=?$ |
| :---: | :---: | :---: | :---: |

## Year 1 Multiplication

Pupils should be taught to:

- solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

| Doubling | Use practical activities to show how to double a number. <br> $4 \times 2=8$ | Draw pictures to show how to double a number. <br> Double 4 is 8 | Show calculation $3 \times 2=6$ |
| :---: | :---: | :---: | :---: |
| Counting in multiples | Count in multiples supported by concrete objects in equal groups. |  | Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, $4,6,8,105,10,15,20$ $25,30$ |
| Repeated addition | Use different objects to add equal groups. |  | Write addition sentences to describe objects and pictures. |

## Year 2 Multiplication

Pupils should be able to:

- recall and use multiplication facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers calculate mathematical statements for multiplication a within the multiplication tables and write them using the multiplication $(\times)$ and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

| Objective and <br> strategy | Concrete | Pictorial | Abstract |
| :--- | :--- | :--- | :--- |
| Doubling | Model doubling using base ten by <br> partitioning tens and ones. Then double the <br> tens and double the ones. Make the total <br> amount. <br> Double 26: <br> $26 \times 2=$ | Draw pictures and representations to <br> show how to double numbers. | $26 \times 2$ |
|  |  |  |  |
|  |  |  |  |


| Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition) | Using different manipulatives to make repeated additions sums. $5+5+5+5+5+5+5+5=40$ $\begin{aligned} & 3+3+3+3=12 \\ & 3 \times 4=12 \end{aligned}$ $\begin{aligned} & 5+5+5+5+5=25 \\ & 5 \times 5=25 \end{aligned}$ | Number lines, counting sticks and bar models should be used to show representation of counting in multiples. | Count in multiples of a number aloud. <br> Write sequences with multiples of numbers. $\begin{aligned} & 0,2,4,6,8,10 \\ & 0,3,6,9,12,15 \\ & 0,5,10,15,20,25,30 \end{aligned}$ $4 \times 3=$ |
| :---: | :---: | :---: | :---: |


| Multiplication is commutative. | Create arrays using counters, numicon and cubes. <br> Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. | Use representations of arrays to show different calculations and explore commutativity. | $\begin{aligned} & 12=3 \times 4 \\ & 12=4 \times 3 \end{aligned}$ <br> Use an array to write multiplication sentences and reinforce repeated addition. $\begin{aligned} & 5+5+5=15 \\ & 3+3+3+3+3=15 \\ & 5 \times 3=15 \\ & 3 \times 5=15 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other. |  |  | $\begin{aligned} & 2 \times 4=8 \\ & 4 \times 2=8 \\ & 8 \div 2=4 \\ & 8 \div 4=2 \\ & 8=2 \times 4 \\ & 8=4 \times 2 \\ & 2=8 \div 4 \\ & 4=8 \div 2 \end{aligned}$ <br> Show all 8 related fact family sentences. |

## Year 3 Multiplication

Pupils should be taught to:

- recall and use multiplication facts for the 3,4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in with $n$ objects are connected to m objects


| Using commutativity to support understanding of the times tables | Understand how to use times-tables facts flexibly. <br> There are 6 groups of 4 pens. <br> There are 4 groups of 6 bread rolls. <br> I can use $6 \times 4=24$ to work out both totals. | Understand how times-table facts relate to commutativity. <br> $\bigcirc 00000$ <br> 000000 <br>  <br> 00000 $\begin{aligned} & 6 \times 4=24 \\ & 4 \times 6=24 \end{aligned}$ |  |  | Understand how times-table facts relate to commutativity. <br> I need to work out 4 groups of 7. <br> I know that $7 \times 4=28$ <br> so, I know that <br> 4 groups of $7=28$ <br> and <br> 7 groups of $4=28$. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Understanding and using $\times 3, \times 2$, $\times 4$ and $\times 8$ tables. | Children learn the times-tables as 'groups of', but apply their knowledge of commutativity. <br> I can use the $\times 3$ table to work out how many keys. <br> I can also use the $\times 3$ table to work out how many batteries. | Children understand how the $\times 2, \times 4$ and $\times 8$ tables are related through repeated doubling. |  |  | Children understand the relationship between related multiplication and division facts in known times-tables. $\begin{aligned} & 2 \times 5=10 \\ & 5 \times 2=10 \\ & 10 \div 5=2 \\ & 10 \div 2=5 \end{aligned}$ |



| Multiplying a 2digit number by a 1-digit number | Understand how to link partitioning a 2-digit number with multiplying. <br> Each person has 23 flowers. <br> Each person has 2 tens and 3 ones. <br> There are 3 groups of 2 tens. <br> There are 3 groups of 3 ones. <br> Use place value equipment to model the multiplication context. <br> There are 3 groups of 3 ones. <br> There are 3 groups of 2 tens. | Use place value is linked with mu number. $3 \times 24=?$  <br> $3 \times 4=12$ $\begin{aligned} & 3 \times 20=60 \\ & 60+12=72 \\ & 3 \times 24=72 \end{aligned}$ | port how partitioning g by a 2 -digit | Use addition to 2-digit numbers $\begin{aligned} & 4 \times 13=? \\ & 4 \times 3=12 \\ & 12+40=52 \\ & 4 \times 13=52 \end{aligned}$ | complete multiplications of by a 1 -digit number. $4 \times 10=40$ <br> The two digit number is partitioned horizontally with the tens digit coming first. The equation is then represented using counters (or an array). <br> Again, the two digit number is partitioned horizontally with the tens digit coming first. This time the equation is represented using place value counters or Dienes. |
| :---: | :---: | :---: | :---: | :---: | :---: |



## Year 4 Multiplication

Pupils should be taught to:

- recall multiplication facts for multiplication tables up to $12 \times 12$
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1 ; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects


## Multiplying by

 multiples of 10 and 100Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.


3 groups of 4 ones is 12 ones. 3 groups of 4 ones is 12 tens. 3 groups of 4 hundreds is 12 hundreds.

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.

$3 \times 40=120$
$3 \times 400=1,200$

Use known facts and understanding of place value and commutativity to multiply mentally.
$4 \times 7=28$
$4 \times 70=280$
$40 \times 7=280$
$4 \times 700=2,800$
$400 \times 7=2,800$

| Understanding times-tables up to $12 \times 12$ | Understand the special cases of multiplying by 1 and 0 . <br> $5 \times 1=5$ <br> $5 \times 0=0$ | Represent the relationship between the $\times 9$ table and the $\times 10$ table, <br> Represent the $\times 11$ table and $\times 12$ tables in relation to the $\times 10$ table. $\begin{aligned} & 2 \times 11=20+2 \\ & 3 \times 11=30+3 \\ & 4 \times 11=40+4 \end{aligned}$ | Understand how times-tables relate to counting patterns. <br> Understand links between the $\times 3$ table, $\times 6$ table and $\times 9$ table $5 \times 6$ is double $5 \times 3$ <br> $\times 5$ table and $\times 6$ table <br> I know that $7 \times 5=35$ <br> so 1 know that $7 \times 6=35+7$. <br> $\times 5$ table and $\times 7$ table $3 \times 7=3 \times 5+3 \times 2$ <br> $\times 9$ table and $\times 10$ table $\begin{aligned} & 6 \times 10=60 \\ & 6 \times 9=60-6 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Understanding and using partitioning in multiplication | Make multiplications by partitioning. <br> $4 \times 12$ is 4 groups of 10 and 4 groups of 2. $4 \times 12=40+8$ | Understand how multiplication and partitioning are related through addition. | Use partitioning to multiply 2-digit numbers by a single digit. $18 \times 6=?$ $\begin{aligned} 18 \times 6 & =10 \times 6+8 \times 6 \\ & =60+48 \\ & =108 \end{aligned}$ |


| Column multiplication for 2- and 3-digit numbers multiplied by a single digit | Use place value equipment to make multiplications. <br> Make $4 \times 136$ using equipment. <br> I can work out how many $1 \mathrm{~s}, 10$ s and 100 s. <br> $\begin{array}{ll}\text { There are } 4 \times 6 \text { ones... } & 24 \text { ones } \\ \text { There are } 4 \times 3 \text { tens } \ldots & 12 \text { tens } \\ \text { There are } 4 \times 1 \text { hundreds ... } 4 \text { hundreds }\end{array}$ $24+120+400=544$ | Use place value equipment alongside a column method for multiplication of up to 3 -digit numbers by a single digit. $\begin{array}{r} 312 \\ \times \quad 3 \\ \hline 936 \\ \hline \end{array}$ | Use the formal column method for up to 3-digit numbers multiplied by a single digit. $\begin{array}{r} 312 \\ \times \quad 3 \\ \hline 936 \\ \hline \end{array}$ <br> Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. <br> 3. Grid method (using place value counters or Dienes) <br> - тохо $\left.\begin{aligned} & 18 \times 3=54 \\ & \times \\ & \hline 3 \end{aligned} \right\rvert\, \begin{aligned} & 10 \\ & \hline \end{aligned}$ <br> The same layout is used as before but this time, the digits are being used. <br> - HTO $\times \mathrm{O}$ <br> The three digit number is partitioned horizontally with the hundreds first followed by the tens and ones. |
| :---: | :---: | :---: | :---: |


| Multiplying more than two numbers | Represent situations by multiplying three numbers together. <br> Each sheet has $2 \times 5$ stickers. <br> There are 3 sheets. <br> There are $5 \times 2 \times 3$ stickers in total. $\begin{aligned} & \underbrace{5 \times 2}_{1} \times 3=30 \\ & 10 \times 3=30 \end{aligned}$ | Understand that commutativity can be used to multiply in different orders. $\begin{array}{r} 2 \times 6 \times 10=120 \\ 12 \times 10=120 \\ 10 \times 6 \times 2=120 \\ 60 \times 2=120 \end{array}$ | Use knowledge of factors to simplify some multiplications. $\begin{aligned} & 24 \times 5=12 \times 2 \times 5 \\ & 12 \times \underbrace{2 \times 5}_{1}= \\ & 12 \times 10=120 \end{aligned}$ <br> So, $24 \times 5=120$ |
| :---: | :---: | :---: | :---: |

## Year 5 Multiplication

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two digit number using a formal written method, including long multiplication for two-digit numbers
- multiply numbers mentally, drawing upon known facts
- multiply whole numbers and those involving decimals by 10,100 and 1,000
- solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving multiplication including understanding the meaning of the equals sign

| Understanding factors | Use cubes or counters to explore the meaning of 'square numbers'. <br> 25 is a square number because it is made from 5 rows of 5 . <br> Use cubes to explore cube numbers. <br> 8 is a cube number. | Use images to explore examples and nonexamples of square numbers. <br> $8^{2}=64$ <br> 12 is not a square number, because you cannot multiply a whole number by itself to make 12. | Understand the pattern of square numbers in the multiplication tables. <br> Use a multiplication grid to circle each square number. Can children spot a pattern? |
| :---: | :---: | :---: | :---: |
| Multiplying by 10,100 and 1,000 | Use place value equipment to multiply by 10,100 and 1,000 by unitising. | Understand the effect of repeated multiplication by 10 . <br> \|||||||||| | Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000. $\begin{aligned} & 17 \times 10=170 \\ & 17 \times 100=17 \times 10 \times 10=1,700 \\ & 17 \times 1,000=17 \times 10 \times 10 \times 10=17,000 \end{aligned}$ |



| Multiplying 2digit numbers by 2-digit numbers | Partition one number into 10 s and 1 s , then add the parts. $23 \times 15=?$ <br>  <br>  <br> Trom <br> $3 \times 15=45$ <br> There are 345 bottles of milk in total. $23 \times 15=345$ | Use an area model $28 \times 15=$ ? $28 \times 15=420$ | add the parts. | Use column multiplication, ensuring understanding of place value at each stage. <br> 5. Long Multiplication (introduced alongside grid method) |
| :---: | :---: | :---: | :---: | :---: |




## Year 6 Multiplication

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operation to carry out calculations involving the 4 operations
- solve problems involving multiplication


| Multiplying up to a 4-digit number by a 2-digit number |  | Use an area model alongside written multiplication. <br> Method I | Use compact column multiplication with understanding of place value at all stages. $\begin{array}{llllll}  & 1 & 2 & 3 & 5 & \\ \times & & 2 & 1 \\ \hline & 1 & 2 & 3 & 5 & \\ & 1 \times 1,235 \\ 2 & 4 & 7 & 0 & 0 & 20 \times 1,235 \\ \hline 2 & 5 & 9 & 3 & 5 & 21 \times 1,235 \\ \hline \end{array}$  |
| :---: | :---: | :---: | :---: |
| Using knowledge of factors and partitions to compare methods for multiplications | Use equipment to understand square numbers and cube numbers. $\begin{aligned} & 5 \times 5=5^{2}=25 \\ & 5 \times 5 \times 5=5^{3}=25 \times 5=125 \end{aligned}$ | Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately. <br> Represent and compare methods using a bar model. | Use a known fact to generate families of related facts. <br> Use factors to calculate efficiently. $\begin{aligned} & 15 \times 16 \\ = & 3 \times 5 \times 2 \times 8 \\ = & 3 \times 8 \times 2 \times 5 \\ = & 24 \times 10 \\ = & 240 \end{aligned}$ |




## Year 1 Division

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher



## Year 2 Division

Pupils should be able to:

- recall and use division facts for the 2,5 and 10
- calculate mathematical statements for multiplication and division
- write the division $(\div)$ and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

| Objectives and strategies | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Division as sharing | I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities. <br> Draw sharing model to support division and halving. <br> Half of 10 is - | $28 \div 7=4$ <br> Divide 28 into 7 groups. How many are in each group? |


| Division as grouping | Division by groupingGroup objects into their divisible factor. | Draw an array and use lines to split the array into groups to make multiplication and division sentences. | Find the inverse of multiplication and division sentences by creating four linking number sentences. $\begin{aligned} & 7 \times 4=28 \\ & 4 \times 7=28 \\ & 28 \div 7=4 \\ & 28 \div 4=7 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Division within arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created. <br> Eg. $\begin{aligned} & 15 \div 3=5 \\ & 5 \times 3=15 \\ & 15 \div 5=3 \\ & 3 \times 5=15 \end{aligned}$ |  |  |


| Division in quarters | Dividing objects into 4 groups and <br> counting the fraction of the amount. | $2 / 4$ of $20=10$ |
| :--- | :--- | :--- | :--- |



## Year 3 Division

Pupils should be taught to:

- recall and use division facts for the 3,4 and 8 multiplication tables
- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which $n$ objects are connected to mobjects

| Using times tables knowledge to divide | Use knowledge of known times-tables to calculate divisions. <br>  <br> 24 divided into groups of 8. There are 3 groups of 8 . | Use knowledge of known times-tables to calculate divisions. <br> 48 divided into groups of 4. <br> There are 12 groups. $\begin{aligned} & 4 \times 12=48 \\ & 48 \div 4=12 \end{aligned}$ | Use knowledge of known times-tables to calculate divisions. <br> I need to work out 30 shared between 5 . <br> I know that $6 \times 5=30$ <br> sol know that $30 \div 5=6$. <br> A bar model may represent the relationship between sharing and grouping. $\begin{aligned} & 24 \div 4=6 \\ & 24 \div 6=4 \end{aligned}$ <br> Children understand how division is related to both repeated subtraction and repeated addition. $24 \div 8=3$ <br> $32 \div 8=4$ |
| :---: | :---: | :---: | :---: |


| Understanding remainders | Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further． <br> ｜IIIIIIIIIIII $\square$ ロ $\qquad$ <br> There are 13 sticks in total． <br> There are 3 groups of 4 ，with 1 remainder | Use images to explain remainders． $22 \div 5=4 \text { remainder } 2$ | Understand that the remainder is what cannot be shared equally from a set． $\begin{aligned} & 22 \div 5=? \\ & 3 \times 5=15 \\ & 4 \times 5=20 \\ & 5 \times 5=25 \ldots \text { this is larger than } 22 \end{aligned}$ $\text { So, } 22 \div 5=4 \text { remainder } 2$ |
| :---: | :---: | :---: | :---: |
| Using known facts to divide multiples of 10 | Use place value equipment to understand how to divide by unitising． <br> Make 6 ones divided by 3. <br> （ロロ）（1） <br> Now make 6 tens divided by 3. <br> What is the same？What is different？ | Divide multiples of 10 by unitising． <br> 12 tens shared into 3 equal groups． 4 tens in each group． | Divide multiples of 10 by a single digit using known times－tables． $180 \div 3=?$ <br> 180 is 18 tens． <br> 18 divided by 3 is 6 ． <br> 18 tens divided by 3 is 6 tens． $\begin{aligned} & 18 \div 3=6 \\ & 180 \div 3=60 \end{aligned}$ |
| 2－digit number divided by 1－ digit number，no remainders | Children explore dividing 2－digit numbers by using place value equipment． <br>  <br> पा川ाय <br> 川1川川 <br> $48 \div 2=$ ？ | Children explore which partitions support particular divisions． | Children partition a number into 10 s and 1 s to divide where appropriate． $\begin{aligned} 60 \div 2 & =30 \\ 8 \div 2 & =4 \\ 30+4 & =34 \end{aligned}$ $68 \div 2=34$ |


|  | First divide the 10 s ． $\square$ ［1111य $\square$ <br> Then divide the 1 s ． $\square$ <br> $0 \square \square$ <br> $0 \square \square \square$ | I need to partition 42 differently to divide by 3. $\begin{aligned} & 42=30+12 \\ & 42 \div 3=14 \end{aligned}$ | Children partition flexibly to divide where appropriate． $\begin{aligned} & 42 \div 3=? \\ & 42=40+2 \end{aligned}$ <br> I need to partition 42 differently to divide by 3. <br> $42=30+12$ <br> $30 \div 3=10$ <br> $12 \div 3=4$ $10+4=14$ $42 \div 3=14$ |
| :---: | :---: | :---: | :---: |
| 2－digit number divided by 1－ digit number，with remainders | Use place value equipment to understand the concept of remainder． <br> Make 29 from place value equipment． Share it into 2 equal groups． <br> There are two groups of 14 and 1 remainder． | Use place value equipment to understand the concept of remainder in division． $29 \div 2=?$ | Partition to divide，understanding the remainder in context． <br> 67 children try to make 5 equal lines． $\begin{aligned} & 67=50+17 \\ & 50 \div 5=10 \\ & 17 \div 5=3 \text { remainder } 2 \\ & 67 \div 5=13 \text { remainder } 2 \end{aligned}$ <br> There are 13 children in each line and 2 children left out． |

## Year 4 Division

Pupils should be taught to:

- recall division facts for multiplication tables up to $12 \times 12$
- use place value, known and derived facts to divide mentally
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

| Understanding the relationship between multiplication and division, including times-tables | Use objects to explore families of multiplication and division facts. $4 \times 6=24$ <br> 24 is 6 groups of 4 . <br> 24 is 4 groups of 6 . <br> 24 divided by 6 is 4 . <br> 24 divided by 4 is 6 . | Represent divisions using an array. $28+7=4$ | Understand families of related multiplication and division facts. <br> I know that $5 \times 7=35$ <br> so I know all these facts: $\begin{aligned} & 5 \times 7=35 \\ & 7 \times 5=35 \\ & 35=5 \times 7 \\ & 35=7 \times 5 \\ & 35 \div 5=7 \\ & 35 \div 7=5 \\ & 7=35 \div 5 \\ & 5=35 \div 7 \end{aligned}$ |
| :---: | :---: | :---: | :---: |



| Dividing 2-digit and 3-digit numbers by a single digit by partitioning into $100 \mathrm{~s}, 10$ s and 1s | Partition into 10s and 1 s to divide where appropriate. <br> $39 \div 3=$ ? $\begin{gathered} 39=30+9 \\ 30+3=10 \\ 9+3=3 \\ 39+3=13 \end{gathered}$ | Partition into $100 \mathrm{~s}, 10$ s and 1 s using Base 10 equipment to divide where appropriate. $39 \div 3=?$ $\begin{gathered} 39=30+9 \\ 30 \div 3=10 \\ 9 \div 3=3 \\ 39 \div 3=13 \end{gathered}$ | Partition into $100 \mathrm{~s}, 10$ s and 1 s using a partwhole model to divide where appropriate. <br> $142 \div 2=$ ? |
| :---: | :---: | :---: | :---: |

## Year 5 Division

Pupils should be taught to:

- divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10,100 and 1,000
- solve problems involving division, including using their knowledge of factors and multiples squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates


## Understanding factors and prime numbers

Understand that prime numbers are numbers with exactly two factors.
$13 \div 1=13$
$13 \div 2=6 r$
$13 \div 4=4 r 1$
1 and 13 are the only factors of 13. 13 is a prime number.

Understand how to recognise prime and composite numbers.
know that 31 is a prime number because can be divided by only 1 and itself without eaving a remainder.

I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.
know that 1 is not a prime number, as it has only 1 factor.

| Understanding inverse operations and the link with multiplication, grouping and sharing | Use equipment to group and share and to explore the calculations that are present. <br> I have 28 counters. <br> I made 7 groups of 4. There are 28 in total. <br> I have 28 in total. I shared them equally into 7 groups. There are 4 in each group. <br> I have 28 in total. I made groups of 4. There are 7 equal groups. | Represent multiplicative relationships and explore the families of division facts. $\begin{aligned} & 60 \div 4=15 \\ & 60 \div 15=4 \end{aligned}$ | Represent the different multiplicative relationships to solve problems requiring inverse operations. <br> $12+3=\square$ <br> $12+\square=3$ $\times 3=12$ $+3=12$  $\times 3$ <br> Understand missing number problems for division calculations and know how to solve them using inverse operations. $\begin{aligned} & 22+?=2 \\ & 22+2=? \\ & ?+2=22 \\ & ?+22=2 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Dividing whole numbers by 10 , 100 and 1,000 | Use place value equipment to support unitising for division. $\begin{aligned} & 4,000 \div 1,000 \\ & 4,000 \text { is } 4 \text { thousands. } \\ & 4 \times 1,000=4,000 \\ & \text { So, } 4,000 \div 1,000=4 \end{aligned}$ | Use a bar model to support dividing by unitising. $380 \div 10=38$ <br> 380 is 38 tens. <br> $38 \times 10=380$ <br> $10 \times 38=380$ <br> So, $380 \div 10=38$ | Understand how and why the digits change on a place value grid when dividing by 10 , 100 or 1,000 . $3,200 \div 100=?$ <br> 3,200 is 3 thousands and 2 hundreds. <br> $200 \div 100=2$ <br> $3,000 \div 100=30$ <br> $3,200 \div 100=32$ <br> So, the digits will move two places to the right. |


| Dividing by multiples of 10,100 and 1,000 | Use place value equipment to represent known facts and unitising. <br> 15 ones put into groups of 3 ones. There are 5 groups. $15 \div 3=5$ <br> 15 tens put into groups of 3 tens. There are 5 groups. $150+30=5$ | Represent related facts with place value equipment when dividing by unitising. <br> 180 is 18 tens. <br> 18 tens divided into groups of 3 tens. There are 6 groups. $180 \div 30=6$ <br> 12 ones divided into groups of 4. There are 3 groups. <br> 12 hundreds divided into groups of 4 hundreds. There are 3 groups. $1200 \div 400=3$ | Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. $\begin{aligned} & 3,000 \div 5=600 \\ & 3,000 \div 50=60 \\ & 3,000 \div 500=6 \\ & 5 \times 600=3,000 \\ & 50 \times 60=3,000 \\ & 500 \times 6=3,000 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Dividing up to four digits by a single digit using short division | Explore grouping using place value equipment. $268 \div 2=?$ <br> There is 1 group of 2 hundreds. <br> There are 3 groups of 2 tens. <br> There are 4 groups of 2 ones. $264 \div 2=134$ | Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting. <br> Lay out the problem as a short division. <br> There is 1 group of 4 in 4 tens. <br> There are 2 groups of 4 in 8 ones. | Use short division for up to 4-digit numbers divided by a single digit. $\begin{array}{r} 05 \quad 5 \quad 6 \\ 7 \longdiv { 3 ^ { 3 } 8 ^ { 3 } q ^ { 4 } 2 } \\ 3,892+7=556 \end{array}$ <br> Use multiplication to check. $556 \times 7=?$ $6 \times 7=42$ $50 \times 7=350$ $500 \times 7=3500$ $3,500+350+42=3,892$ |


|  |  | Work with divisions that require exchange. |  |
| :---: | :---: | :---: | :---: |
| Understanding remainders | Understand remainders using concrete versions of a problem. <br> 80 cakes divided into trays of 6 . $\square$ <br> 80 cakes in total. They make 13 groups of 6 , with 2 remaining. | Use short division and understand remainders as the last remaining 1 s . | In problem solving contexts, represent divisions including remainders with a bar model. $\begin{aligned} & 683=136 \times 5+3 \\ & 683 \div 5=136 r 3 \end{aligned}$ |


| Dividing decimals by 10,100 and 1,000 | Understand division by 10 using exchange. <br> 2 ones are 20 tenths. <br> 20 tenths divided by 10 is 2 tenths. | Represent division using exchange on a place value grid. <br> 1.5 is 1 one and 5 tenths. <br> This is equivalent to 10 tenths and 50 hundredths. <br> 10 tenths divided by 10 is 1 tenth. 50 hundredths divided by 10 is 5 hundredths. <br> 1.5 divided by 10 is 1 tenth and 5 hundredths. $1 \cdot 5 \div 10=0.15$ | Understand the movement of digits on a place value grid. $0.85 \div 10=0.085$ $8.5 \div 100=0.085$ |
| :---: | :---: | :---: | :---: |
| Understanding the relationship between fractions and division | Use sharing to explore the link between fractions and division. <br> 1 whole shared between 3 people. Each person receives one-third. <br>  | Use a bar model and other fraction representations to show the link between fractions and division. $1 \div 3=\frac{1}{3}$ | Use the link between division and fractions to calculate divisions. $\begin{aligned} & 5 \div 4=\frac{5}{4}=1 \frac{1}{4} \\ & 11 \div 4=\frac{11}{4}=2 \frac{3}{4} \end{aligned}$ |

## Year 6 Division

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations

Understanding factors
Use equipment to explore different factors
of a number.
4 is a factor of 24 but is not a factor of 30 .


Recognise and know primes up to 100. Understand that 2 is the only even prime and that 1 is not a prime number.

| 1 | (2) (3) | 4 | (5) | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| (II) | 12 | 14 | 15 | 16 | 17 | 18 | (19) | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 22 |  |  |  |  |  |  |  |


| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | $(29)$ | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 32 | 3 | 3 | 35 | 3 | 37 | 38 |  |  |


| (31) | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



| Dividing by a single digit | Use equipment to make groups from a total. <br> 09909990909 <br> 99099999 <br> 0909090990909 <br> There are 78 in total. <br> There are 6 groups of 13 <br> There are 13 groups of 6 . |  | Use short division to divide by a single digit. $6 \longdiv { 1 1 3 2 }$ $6 \longdiv { 0 2 }$ $\begin{array}{r} 022 \\ 6 \longdiv { 1 1 3 1 2 } \end{array}$ <br> Use an area model to link multiplication and division. <br> 6 $132+6=20+2=22$ |
| :---: | :---: | :---: | :---: |
| Dividing by a 2-digit number using factors | Understand that division by factors can be used when dividing by a number that is not prime. | Use factors and repeated division. $1,260 \div 14=?$ $\square$ <br> 1.260 $\square$ <br> $1,260 \div 2=630$ $\begin{aligned} & 630 \div 7=90 \\ & 1,260 \div 14=90 \end{aligned}$ | Use factors and repeated division where appropriate. |


| Dividing by a 2-digit number using long division | Use equipment to build numbers from groups. <br>  <br> 182 divided into groups of 13. <br> There are 14 groups. | Use an area model alongside written division to model the process. <br> $377 \div 13=29$ | Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). <br> Write the required multiples to support the division process. <br> $377 \div 13=$ ? <br> $1 3 \longdiv { 3 7 7 }$ <br> $-\begin{array}{r}130 \\ \hline 247\end{array}$ <br> $-$130 <br> 137 <br> $-117 \frac{9}{29}$ <br> $377 \div 13=29$ <br> A slightly different layout may be used, with the division completed above rather than at the side. <br> Divisions with a remainder explored in problem-solving contexts. |
| :---: | :---: | :---: | :---: |


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| Dividing by 10, 100 and 1,000 | Use place value equipment to explore division as exchange. <br> 0.2 is 2 tenths. <br> 2 tenths is equivalent to 20 hundredths. 20 hundredths divided by 10 is 2 hundredths. | Represent division to show the relationship with multiplication. Understand the effect of dividing by 10,100 and 1,000 on the digits on a place value grid. <br> Understand how to divide using division by 10,100 and 1,000 . $12 \div 20=?$ $\qquad$ $\square$ $\prod_{12+10=12}$ | Use knowledge of factors to divide by multiples of 10,100 and 1,000 . $40 \div 50=$ $\square$ $\begin{aligned} & 40 \rightarrow+\div 10 \rightarrow+5 \\ & 40 \rightarrow+\div 5 \end{aligned}$ $40 \div 5=8$ $8 \div 10=0.8$ <br> So, $40 \div 50=0.8$ |


| Dividing decimals | Use place value equipment to explore division of decimals. $\square$ $\square$ $\square$ <br> 8 tenths divided into 4 groups. 2 tenths in each group. | Use a $\square$ <br> ? <br> $4 \times 2=$ <br> So, $4 \times$ | mod <br>  <br> $?$ <br> 0 <br> $=0.8$ |  | epresent divisions. $\square$ <br> $?$ <br> $=2$ <br> $4=0.2$ | Use short division to divide decimals with up to 2 decimal places. $\begin{gathered} 8 \longdiv { 4 \cdot 2 4 } \\ 8 \longdiv { 4 \cdot 4 2 4 } \\ 0 \cdot 5 \\ 8 \longdiv { 4 \cdot 4 ^ { 2 } 4 } \\ 0 \cdot 5 \frac{4}{4} \\ 8 \longdiv { 4 \cdot 4 ^ { 2 } 4 } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

## Year 3 Fractions

## Pupils should be taught to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit number or quantities by 10 .
- recognise, find and write fractions of a discrete set of objects, unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers, unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole for example, 5/7 + 1/7 = 6/7
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above

Commented [1]: Sorry!!! CJ asked me to add in a
fractions bit too :
We can do it for just ks2?




| Recognise that tenths arise from dividing an object into 10 equal parts and in dividing onedigit numbers or quantities by 10 | 0    <br> 0    <br> Use tens frames to represent tenths and count in tenths. Could also use a ten piece from numicon set with an object into the circles to represent the amount of tenths. <br> Using 10 p coins with 10 adding up to $£ 1$ also links to the decimal place. <br> How many tenths of a whole pound do you have? $3 / 10$ link to how it would be written as money $£ 0.30$. | Which tenth is represented by the letter a? B? C? D? E? <br> Eight tenths $\frac{7}{10}$ $\square$ | What do you notice? <br> $1 / 10$ of $10=1$ <br> $2 / 10$ of $10=2$ <br> $3 / 10$ of $10=3$ <br> Continue the pattern. <br> What do you notice? <br> What about $1 / 10$ of 20 ? Use this to work <br> out ${ }^{2} / 10$ of 20 , etc. <br> $1 / 10$ of $100=10$ <br> $1 / 100$ of $100=1$ <br> $2 / 10$ of $100=20$ <br> $2 / 100$ of $100=2$ <br> How can you use this to work out ${ }^{6} / 10$ of 200 ? ${ }^{6} / 100$ of 200? |
| :---: | :---: | :---: | :---: |




## Year 4 Fractions

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to $1 / 4,1 / 2,3 / 4$
- find the effect of dividing a one- or two-digit number by 10 and 100 , identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

| Counting up and down in hundredths. | Use 100 bead strings to rehearse counting forwards and backwards. Use different starting points and draw discussion into the fact that ${ }^{1} / 10$ $={ }^{10} / 100$. | As for counting in tenths Use dienes units blocks to represent 1/100 <br> Use pennies to recognise $1 / 100$ of a pound etc |  |  |  |  |  |  |  |  |  | As for counting in tenths <br> 9.8 is equivalent to 980 hundredths, 9.82 is equivalent to ? hundredths. How many hundredths are there in 10.0? <br> Using a partially filled in hundredths square, ask how you know where e.g. 2.76 would go. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.3 | 2.5 | 2.5 |  |  |  |
|  | (10) | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.4 | 2 | 3 |  |  |  |
|  | To start with, use the hundredths counters to embed the idea that 10 hundredths $=1$ tenth. | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 0.49 | 0.5 |  |  |  |  | 2.6 |
|  | Children can then exchange their ten hundredths | 051 | 0.52 | 0.53 | 0.54 | 0.55 | 0.56 | 0.57 | 0.58 | 0.59 | 0.6 |  |  |  |  |  |
|  | for a tenth counter. Start at different points to rehearse counting forwards and backwards. | 0.61 | 0.62 | 0.63 | 0.64 | 0.65 | 0.66 | 0.67 | 0.68 | 0.69 | 0.7 |  |  |  |  |  |
|  |  | 0.71 | 0.72 | 0.73 | 0.74 | 0.75 | 0.76 | 0.77 | 0.78 | 0.79 | 0.8 |  |  | 2.8 | 2.8 |  |
|  |  | 0.81 | 0.82 | 0.83 | 0.84 | 0.85 | 0.86 | 0.87 | 0.88 | 0.89 | 0.9 | 2.8 2 |  |  |  |  |
|  |  | 0.91 | 0.92 | 0.93 | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 | 1 |  |  |  | 2.9 6 |  |


| Recognise and write decimal equivalents of any number of tenths or hundredths |  | $\frac{60}{100}$ <br> $\frac{70}{100}$ <br> $\frac{8}{10}$ | 0.60 <br> 0.70 <br> 0.8 |  | What do you notice? <br> One tenth of $£ 41$ <br> One hundredth of $£ 41$ <br> One thousandth of $£ 41$ Continue the pattern. What do you notice? $\begin{aligned} & 0.085+0.015=0.1 \\ & 0.075+0.025=0.1 \\ & 0.065+0.035=0.1 \end{aligned}$ <br> Continue the pattern for the next five number sentences |
| :---: | :---: | :---: | :---: | :---: | :---: |




## Year 5 Fractions

## Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same numbers
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements $>1$ as a mixed number [for example, $52+54=56=151$
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams read and write decimal numbers as fractions [for example, 0.71 = 10071 ]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalent
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the percent symbol (\%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of $1 / 2,1 / 4,1 / 5,2 / 5,4 / 5$ and those fractions with a denominator of a multiple of 10 or 25 .





## Year 6 Fractions

## Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 41×21=81]
- divide proper fractions by whole numbers [for example, $31 \div 2=61$ ]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.


Add and subtract fractions with Once they have combined their previous knowledge of different denominators


Add and subtract fractions, including mixed numbers and improper fractions where bridging is required

Additio
Example: $3 \frac{5}{7}+2 \frac{4}{7}$
Use the same method as previously with the strips, but get the children to cut up the fraction elements into the component sevenths. This would leave 5 wholes ( 5 strips of $\frac{7}{7}$ ) and nine $\frac{1}{7}$ parts. Elicit that nine $\frac{1}{7}$ parts is the same as $\frac{9}{7}$ or $1 \frac{2}{7}$ parts. Combine 5 with $1 \frac{2}{7}$ to get the answer of $6 \frac{2}{7}$

## Subtraction

Example: $4 \frac{2}{7}-2 \frac{5}{7}$
Use the same method as previously with the strips, but get the children to cut up the fraction elements
into the component sevenths.
into the component sevenths.
This would leave 2 wholes ( 2 strips of $\frac{7}{7}$ ) but then they
would realise that they cannot take $\frac{5}{7}$ away from $\frac{2}{7}$. Discuss what could be done next.
Elicit that they could cut one of their two wholes into $\frac{7}{7}$ Going back to their original $4 \frac{2}{7}$ they would the repartition that into $3 \frac{9}{7}$.
With $3 \frac{9}{7}$ in strips, they could remove the 2 wholes then the $\frac{4}{7}$ leaving them with $1 \frac{5}{7}$.


(

| Divide proper fraction by whole numbers. | Using paper strips is a good way to demonstrate what happens to a fraction when it is divided by a whole integer i.e. 2 <br> Fold the paper in half: What happens when I divide $1 / 2$ by 2? By folding each half into two parts, it becomes clear that quarters have been formed. | Lee has $\frac{2}{5}$ of a chocolate bar. He shares it with his friend. How much chocolate do they get each? $\square$ <br> Use the diagrams to help you calculate. <br> Calculate the following and use the diagram to help you. $\frac{1}{8} \div 4=$ | Solve: $\left[\begin{array}{l} 6 \div-=9 \\ -\div \frac{2}{5}=10 \\ -\div-=5 \frac{5}{6} \end{array}\right.$ <br> Harjoht's Mum ordered pizza for the whole family. Harjoht ate $1 / 4$ of the pizza. His Mum, brother and sister ate the rest of the pizza. What fraction of the pizza did they get each? <br> Molly's Mum ordered pizza for the whole family. Molly ate $1 / 4$ of the pizza. Six members of the family eat the remaining pizza. What fraction of the pizza did they get each? |
| :---: | :---: | :---: | :---: |

## Vocabulary

## Addition \& Subtraction

Years 3 and 4: add, addition, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? how many more to make...? how many more is... than...? how much more is...? -, subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? how many fewer is... than...? how much less is....? difference between, half, halve, how many more/fewer is... than...? how much more/less is...? Is equal to, is the same as, tens boundary, hundreds boundary, inverse

Years 5 and 6: add, addition, more, plus, increase, sum, total, altogether, double, near double, how many more to make...? subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is... than...? how much more/less is...? Is equal to, sign, is the same as, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse

## Multiplication \& Division

Year 3 and 4: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product, once, twice, three times... ten times...times as (big, long, wide... and so on), repeated addition, array, row, column, double, halve, share, share equally, one each, two each, three each...group in pairs, threes... tens, equal groups of, divide, division, divided by, divided into, remainder, factor, quotient, divisible by, inverse

Years 5 and 6: lots of, groups of, times, multiply, multiplication, multiplied by, multiple of, product once, twice, three times... ten times...times as (big, long, wide... and so on), repeated addition array, row, column, double, halve, share, share equally, one each, two each, three each...group in pairs, threes... tens, equal groups of, divide, division, divided by, divided into, dividend, divisor, remainder, factor, quotient, divisible by, inverse, fraction

## Reasoning and Problem Solving

All children to be given the opportunity to reason and problem solve at least once a week.
Children should be using the Answer Prove it Explain it guidelines when reasoning.

See Reasoning Examples for further guidance.

